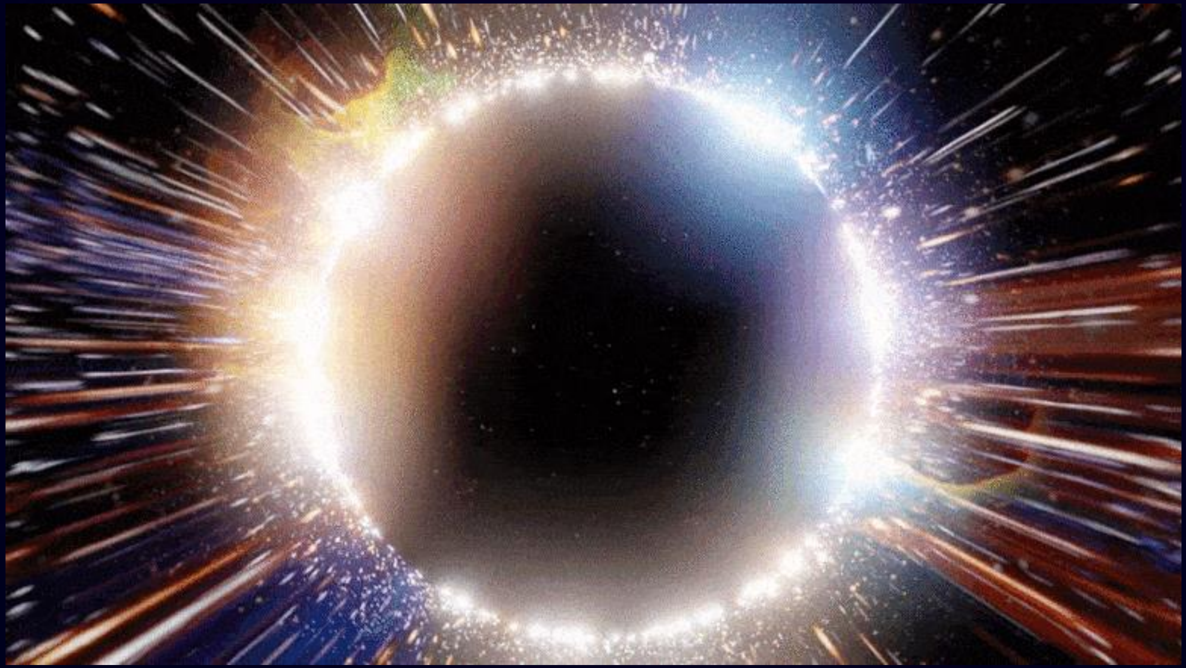
In the name of God

Black hole thermodynamics in modified gravitational theories



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Supervisor: Prof.Mehrdad Farhoudi

Classical thermodynamics of black holes in general relativity

Gravitational decoupling and modified gravitational theories

Thermodynamics of deformed AdS-Schwarzschild black hole

Results and discussions

Outlook

Four lows of black hole mechanics

0 The surface gravity κ is constant over the event horizon of stationary black hole.

1
$$\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega\delta J + \Phi\delta Q$$
(1)

$$\delta U = T\delta S - P\delta V + \sum_j \mu_j \delta N_j + \Phi\delta Q$$
(2)

κ: surface gravity
Ω: angular velocity
Φ:electric potential

2 The area of a black hole's event horizon can never decrease ,($\delta A \geq 0$).

3 It is impossible to reduce the surface gravity κ to zero in finite number of steps.

J.M. Bardeen, B. Carter and S.W. Hawking, "The four laws of black hole mechanics", *Commun. Math. Phys.* **31** (1973) 161.

Basic questions in black hole thermodynamics

Black hole temperature?

S.W. Hawking, "Particle creation by black holes", *Commun. Math. Phys.* **43** (1975) 199.

$$k_B T = \frac{\hbar \kappa}{2\pi c} \quad (3)$$

Black hole entropy?

J.D. Bekenstein, "Generalized second law of thermodynamics in black-hole physics", *Phys. Rev. D* **912** (1974) 3292.

$$S = \frac{Ac^3}{4\hbar G} \quad (4)$$

$$\frac{\kappa}{8\pi G}\delta A \Rightarrow T\delta S \quad (5)$$

Basic questions in black hole thermodynamics

$$\delta M = T\delta S + \Omega\delta J + \Phi\delta Q \quad (6)$$

Absence of $P\delta V$ term

C. Teitelboim, "The cosmological constant as a thermodynamic black hole parameter", *Phys. Lett. B* **158** (1985) 293.

J.D. Brown and C. Teitelboim, "neutralization of the cosmological constant by membrane creation", *Nucl. Phys. B* **297** (1988) 787.

L. Smarr, "Mass formula for Kerr black holes", *Phys. Rev. Lett.* **30** (1973) 71.

$$M = 2\,(TS + \Omega J) + \Phi Q \quad (7)$$

B.P. Dolan, "The cosmological constant and the black hole equation of state", *Class.Quant. Grav.* **28** (2011) 125020.

$$(d-3)M = (d-2)\frac{\partial M}{\partial A}A + (d-2)\sum_i \frac{\partial M}{\partial J^i}J^i + (-2)\frac{\partial M}{\partial \Lambda}\Lambda + (d-3)\sum_j \frac{\partial M}{\partial \Phi^j}\Phi^j \quad (8)$$

$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2} \quad (9)$$

d: Dimension of space time

$$\delta M = T\delta S + \Omega\delta J + \Phi\delta Q$$

$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2} \qquad \textbf{(10)}$$

Volume as the thermodynamic dual of pressure

T. Padmanabhan, "Classical and quantum thermodynamics of horizons in spherically symmetric space-times", *Class. Quant. Grav.* **19** (2002) 5387.

$$\delta M = T\delta S + V\delta P + \Omega\delta J + \Phi\delta Q \qquad \textbf{(11)}$$

$$V \equiv \left(\frac{\partial M}{\partial P}\right)_{S,Q,J}$$

The classical thermodynamics of black hole in canonical ensemble

- حل معادلات حرکت و یافتن پاسخ سیاه‌چاله
- محاسبه متغیرهای ...

$$g^{rr}|_{r_h} = 0 \qquad T = \frac{1}{4\pi}\frac{\partial}{\partial r} g^{rr}|_{r_h} \qquad S = \frac{A_h}{4} \qquad P = -\frac{\Lambda}{8\pi} \qquad (12)$$

- محاسبه پتانسیل های ترمودینامیکی

$$C_P \equiv C_{P,J_i,Q_j} = T\left(\frac{\partial S}{\partial T}\right)_{P,J_i,Q_j}$$

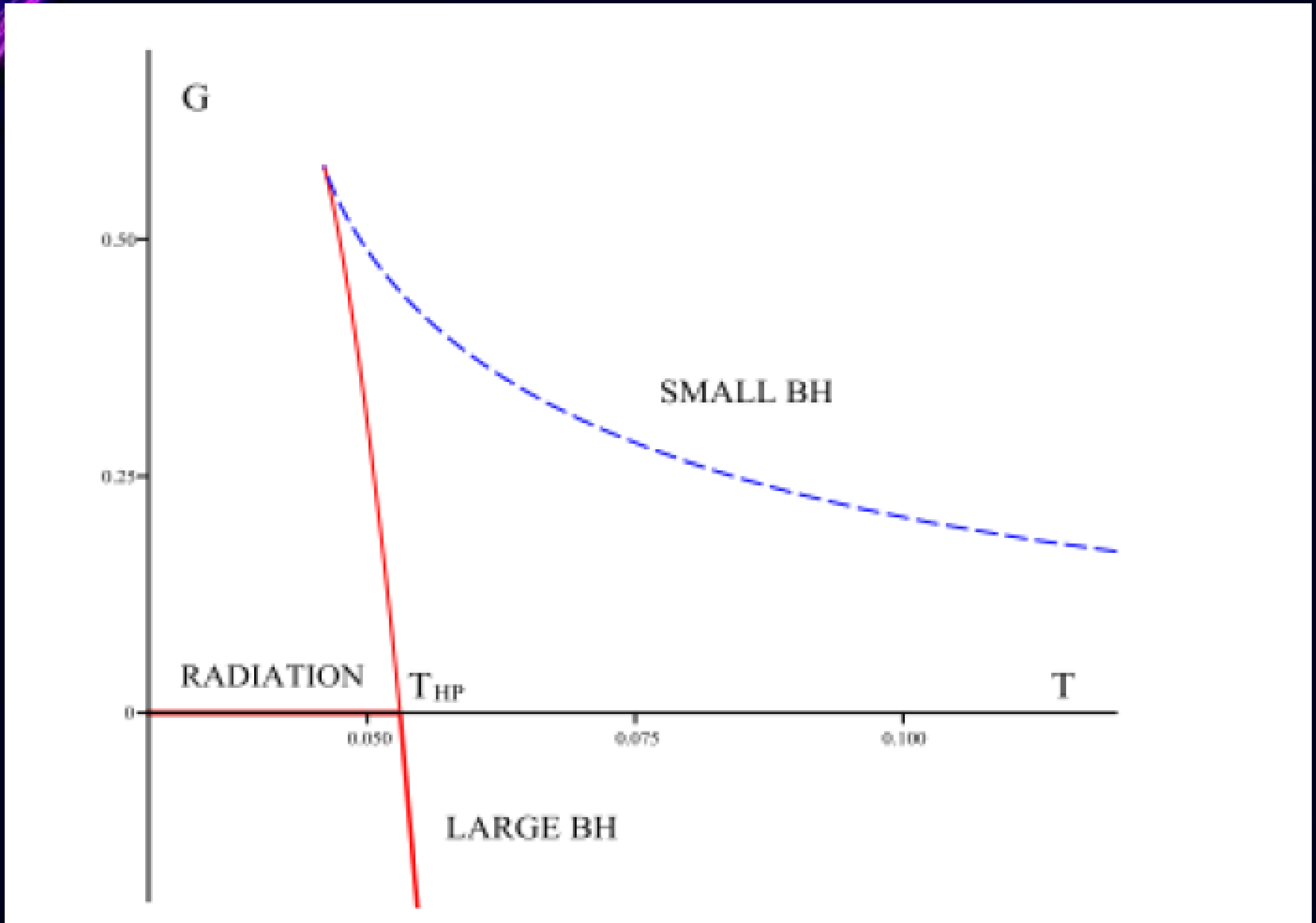
$$G = M - TS = G(P,T,J_i,Q_j) \qquad (13)$$

- تعیین گذار فازهای ترمودینامیکی

D. Kubiznak and R.B. Mann, "Black hole chemistry", *Can. J.Phys.* **93** (2015) 999.

A. Karch and B. Robinson, "Holographic black hole chemistry", *J. High Energy Phys.* **12** (2015) 073.

[106] S.W. Hawking and D.N. Page, "Thermodynamics of black holes in anti-De Sitter Space", *Commun. Math. Phys.* **87** (1983) 577.



E. Witten, "Anti-de Sitter space and holography", *Adv. Theor. Math. Phys.* **2** (1998) 253.

Black hole thermodynamics?

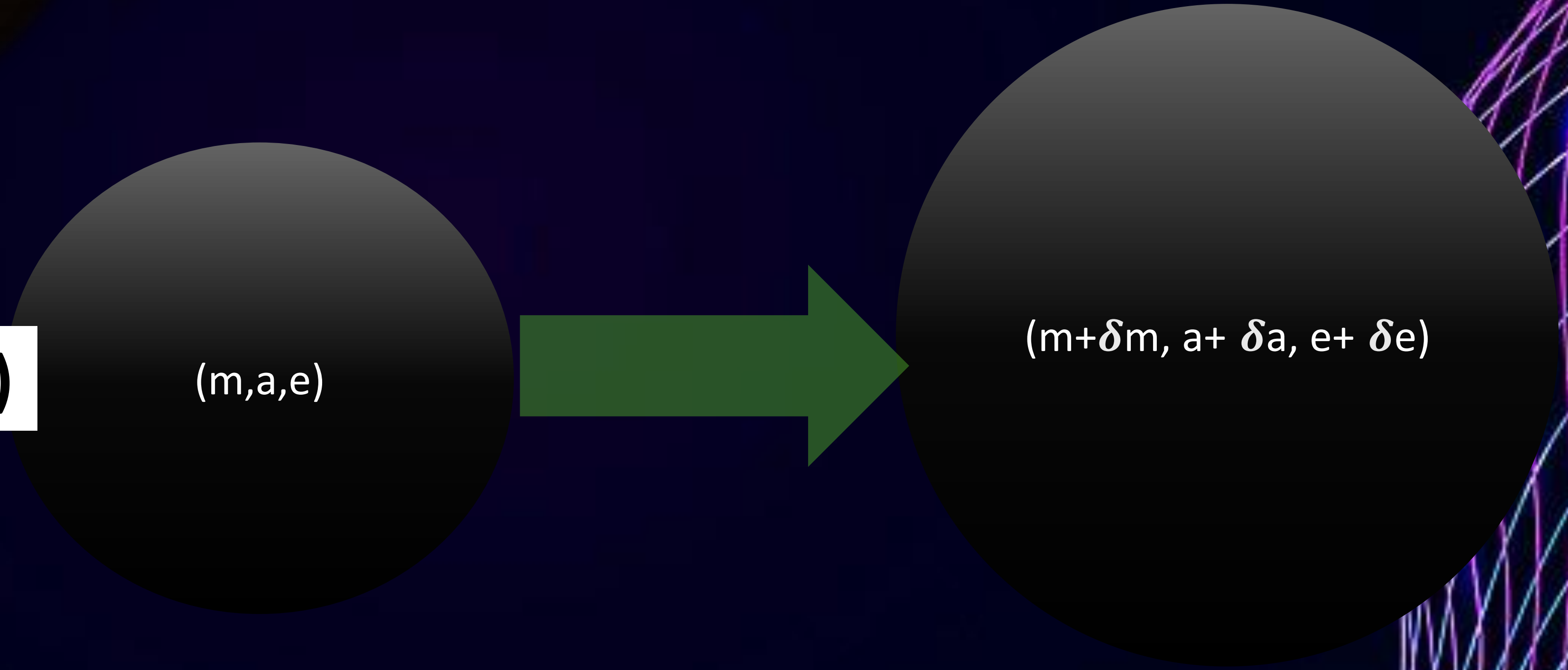
$$\delta M = T\,\delta S + \Omega\,\delta J + \Phi\,\delta Q + V\,\delta P \qquad (14)$$

R.M. Wald, "Black hole entropy is the Noether charge", *Phys. Rev.D* **48** (1993) 3427.

Kerr- Newman

M=m
J=ma
Q=e

$$T = \frac{1}{4\pi}F'(r_+)$$
$$\Omega = \frac{a}{r_+^2 + a^2}$$
$$\Phi = \frac{e r_+}{r_+^2 + a^2} \qquad (15)$$



Black hole thermodynamics in modified gravitational theories and Conservation of the structure of the AdS- Schwarzschild black hole as our approach .

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Regular Article - Theoretical Physics

Thermodynamics of deformed AdS-Schwarzschild black hole

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Our method :Gravitational decoupling

J. Ovalle and F. Linares, "Tolman IV solution in the Randall-Sundrum braneworld", *Phys. Rev. D* **88** (2013) 104026.

J. Ovalle, R. Casadio, R. da Rocha, A. Sotomayor and Z. Stuchlik, "Black holes by gravitational decoupling", *Eur. Phys. J. C* **78** (2018) 960.

E. Contreras, J. Ovalle and R. Casadio, "Gravitational decoupling for axially symmetric systems and rotating black holes", *Phys. Rev. D* **103** (2021) 044020.

Gravitational decoupling method

$$A = \int d^4x \sqrt{-g}\left(\frac{R-2\Lambda}{2\kappa} + L_m + L_X\right) \quad \kappa = 8\pi G_N/c^4 \qquad (16)$$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{(\mathrm{tot})}_{\mu\nu} \qquad (17)$$

$$T^{(\mathrm{tot})}_{\mu\nu} = T^{(\mathrm{m})}_{\mu\nu} + T^{(\mathrm{X})}_{\mu\nu} \qquad (18)$$

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\mu(r)}dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2) \qquad (19)$$

] J. Ovalle, "Decoupling gravitational sources in general relativity: The extended case", *Phys. Lett. B* **788** (2019) 213.

Gravitational decoupling method-EOMs

$$\kappa\left(T^{(\mathrm{m})0}_{\ 0}+T^{(\mathrm{X})0}_{\ 0}\right)=\Lambda+e^{-\mu}\left(\frac{1}{r^2}-\frac{\mu'}{r}\right)-\frac{1}{r^2},$$

$$\kappa\left(T^{(\mathrm{m})1}_{\ 1}+T^{(\mathrm{X})1}_{\ 1}\right)=\Lambda+e^{-\mu}\left(\frac{1}{r^2}+\frac{\nu'}{r}\right)-\frac{1}{r^2},$$

$$\kappa\left(T^{(\mathrm{m})2}_{\ 2}+T^{(\mathrm{X})2}_{\ 2}\right)=\Lambda+\frac{1}{4}e^{-\mu}\left[-\mu'\nu'+2\nu''+\nu'^2+\frac{2\left(\nu'-\mu'\right)}{r}\right],$$

(20)

$$\left(T^{(\mathrm{m})}{}_{\mu}{}^{\nu}\right)=\mathrm{diag}[-\epsilon,p_r,p_t,p_t]$$

$$\left(T^{(\mathrm{X})}{}_{\mu}{}^{\nu}\right)=\mathrm{diag}[-\mathcal{E},\mathcal{P}_r,\mathcal{P}_t,\mathcal{P}_t]$$

$$\left(T^{(\mathrm{tot})}{}_{\mu}{}^{\nu}\right)=\mathrm{diag}[-\tilde{\epsilon},\tilde{p}_r,\tilde{p}_t,\tilde{p}_t]$$

(21)

Gravitational decoupling method-The first assumptions

$$ds^2\big|_{T^{(\mathrm{X})}_{\mu\nu}=0} = -e^{\zeta(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + sin^2\theta\, d\phi^2) \quad \textbf{(22)}$$

$$\zeta \rightarrow \nu = \zeta + \alpha\, g(r)$$

$$e^{-\lambda} \rightarrow e^{-\mu} = e^{-\lambda} + \alpha\, f(r) \quad \textbf{(23)}$$

$$\kappa\,\epsilon = -\Lambda - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}$$

$$\kappa\, p_r = \Lambda + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\zeta'}{r}\right) - \frac{1}{r^2}$$

$$\kappa\, p_t = \Lambda + \frac{1}{4}e^{-\lambda}\left[-\lambda'\zeta' + 2\zeta'' + \zeta'^2 + \frac{2\,(\zeta' - \lambda')}{r}\right]$$

$$\kappa\,\mathcal{E} = -\frac{\alpha\, f}{r^2} - \frac{\alpha\, f'}{r}$$

$$\kappa\,\mathcal{P}_r - \alpha\frac{e^{-\lambda}g'}{r} = \alpha f\left(\frac{1}{r^2} + \frac{\nu'}{r}\right)$$

$$\kappa\,\mathcal{P}_t - \frac{\alpha e^{-\lambda}}{4}\left(2g'' + \alpha g'^2 + \frac{2g'}{r} + 2g'\zeta' - \lambda' g'\right) = \frac{\alpha f}{4}\left(2\nu'' + \nu'^2 + 2\frac{\nu'}{r}\right) + \frac{\alpha f'}{4}\left(\nu' + \frac{2}{r}\right).$$

$$\nabla_\mu T^{(\mathrm{m})\,\mu}{}_{\nu} = -\frac{\alpha\, g'}{2}(\epsilon + p_r)\delta_{1\,\nu} = -\nabla_\mu T^{(\mathrm{X})\,\mu}{}_{\nu}$$

(24) **(25)**

Gravitational decoupling method- Secondary assumptions

$$e^{\nu(r)} = e^{-\mu(r)} \quad (26)$$

$$e^{\zeta(r)}\big|_{T^{(\mathrm{m})}_{\mu\nu}=0} = e^{-\lambda(r)}\big|_{T^{(\mathrm{m})}_{\mu\nu}=0} = 1 - \frac{2M}{r} + \frac{r^2}{l^2} \quad (27)$$

Preliminary results

$$\nabla_\mu T^{(\mathrm{m})\,\mu}{}_{\nu} = -\frac{\alpha\, g'}{2}(\epsilon + p_r)\delta_{1\,\nu} = -\nabla_\mu T^{(\mathrm{X})\,\mu}{}_{\nu} \quad \text{(28)}$$

$$ds^2 = -e^{\varsigma(r)}B(r)dt^2 + \frac{1}{e^{\varsigma(r)}B(r)}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \quad \text{(29)}$$

$$\nu(r) = Ln\left[B(r)\left(1 - \frac{2M}{r} + \frac{r^2}{l^2}\right)\right] \qquad B(r) \equiv e^{\alpha\, g(r)} \quad \text{(30)}$$

$$r^2\kappa\,\mathcal{E} = \left(2M - r - \frac{r^3}{l^2}\right)B'(r) + \left(1 + \frac{3r^2}{l^2}\right)[1 - B(r)] \quad \text{(31)}$$

Gravitational decoupling method in brief

$$A = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{2\kappa} + L_X \right) \quad (32)$$

$$G^{\mu}{}_{\nu} + \Lambda g^{\mu}{}_{\nu} = \kappa \, T^{(X)\mu}{}_{\nu} \quad (33)$$

$$ds^2 = -e^{\zeta(r)} B(r) dt^2 + \frac{1}{e^{\zeta(r)} B(r)} dr^2 + r^2 (d\theta^2 + sin^2\theta d\phi^2) \quad (34$$

$$-r^2 \kappa \, T^{(X)\,0}{}_{0} = \left(2M - r - \frac{r^3}{l^2}\right) B'(r) + \left(1 + \frac{3r^2}{l^2}\right) [1 - B(r)] \quad (35)$$

Gravitational decoupling method overall discussion

$$r^2\kappa\,\mathcal{E} = \left(2M - r - \frac{r^3}{l^2}\right)B'(r) + \left(1 + \frac{3r^2}{l^2}\right)[1 - B(r)]$$

(36)

$$B(r) = \frac{C_0 + r^2 l + r^3 - \kappa\, l^2 \int_\beta^r x^2\mathcal{E}(x)\,dx}{l^2(r-2M) + r^3}$$

$$m(r) = \int_\beta^r x^2\mathcal{E}(x)\,dx$$

(37)

$$e^{\zeta(r)}B(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2} - \frac{\kappa\, m(r)}{r}$$

$$M \to M(r) = M + \frac{\kappa}{2}m(r)$$

(38)

Determining the model for energy density
Step 1

$$r^2\kappa\mathcal{E} = \left(2M - r - \frac{r^3}{l^2}\right)B'(r) + \left(1 + \frac{3r^2}{l^2}\right)[1 - B(r)] \quad \textbf{(39)}$$

WEC

$$T^{\mu}{}_{\nu} = diag(\rho, p_1, p_2, p_3) \quad \textbf{(40)}$$

$$\rho \geq 0, \qquad \rho + p_i \geq 0, \quad i = 1, 2, 3$$

$$\mathcal{E} \geq 0 \quad \textbf{(41)}$$

$$2(\mathcal{E} + \mathcal{P}_t) = -r\mathcal{E}' \geq 0 \quad \textbf{(42)}$$

Determining the model for energy density
Step 2

$$L_X \propto R^2 \Longrightarrow \epsilon \propto \frac{1}{r^4}$$ **(43)**

$$\Longrightarrow \epsilon(r) \propto \frac{1}{r^4}$$ **(45)**

$$L_X \propto \frac{1}{R} \Longrightarrow \epsilon \propto \frac{1}{r^4} + constant$$ **(44)**

Modeling the problem

$$r^2\kappa\,\mathcal{E} = \left(2M - r - \frac{r^3}{l^2}\right)B'(r) + \left(1 + \frac{3r^2}{l^2}\right)[1 - B(r)] \tag{46}$$

$$-T^{(X)\,0}{}_{0} \equiv \mathcal{E}(r) = \frac{\alpha}{\kappa(\beta + r)^4} \tag{47}$$

$$B(r) = \frac{\frac{\alpha\beta^2}{3(\beta+r)^3} + \frac{\alpha}{\beta+r} - \frac{\alpha\beta}{(\beta+r)^2} + r + \frac{r^3+c_1}{l^2}}{r(1 - 2M/r + r^2/l^2)} \tag{48}$$

$$e^{\zeta(r)}B(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2} + \alpha\frac{\beta^2 + 3r^2 + 3\beta r}{3r(\beta + r)^3} \equiv F(r) \tag{49}$$

Deformed AdS-Schwarzschild black hole

Discussion 1- Topological equivalence of the solution with the black hole of Einstein-Maxwell theory

$$e^{\zeta(r)}B(r) = 1 - \frac{2M}{r} + \frac{r^2}{l^2} + \alpha\frac{\beta^2 + 3r^2 + 3\beta r}{3r(\beta + r)^3} \equiv F(r) \quad (50)$$

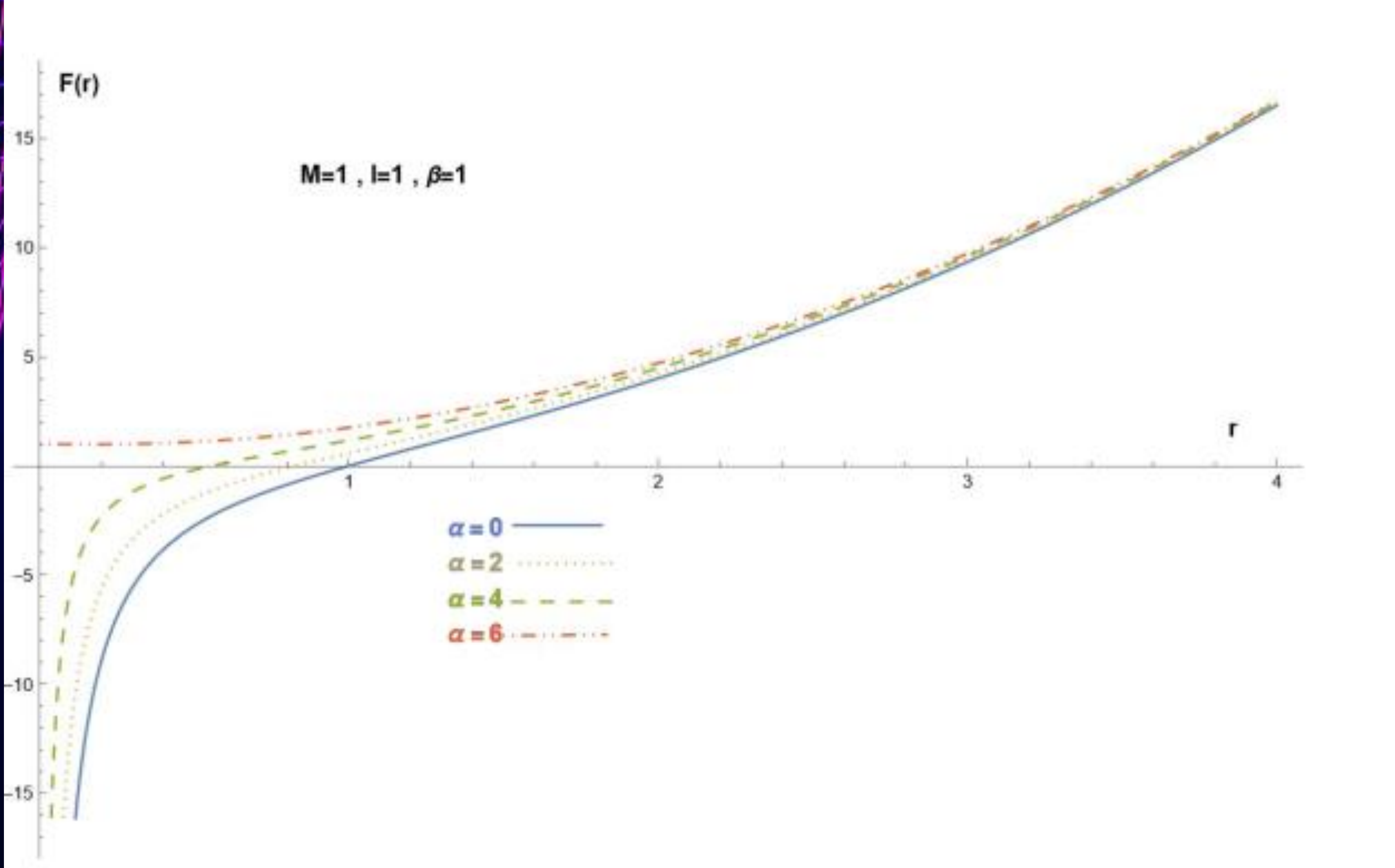
$$F(r)|_{\beta\to 0} = 1 - \frac{2M}{r} + \frac{r^2}{l^2} + \frac{\alpha}{r^2} \quad (51)$$

$$\mathcal{P}_{\text{eff}} \equiv \frac{1}{3}(\mathcal{P}_r + \mathcal{P}_t + \mathcal{P}_t) = \frac{1}{3}\mathcal{E} \quad (52$$

$$T^{(\mathrm{X})} = -\frac{4\beta}{\beta + r}\mathcal{E} \quad (53)$$

H. Abdusattar, "Stability and Hawking-Page-like phase transition of phantom AdS black holes", *Eur. Phys. J. C* **83** (2023) 614.

Discussion 2- Reducing the radius of the event horizon

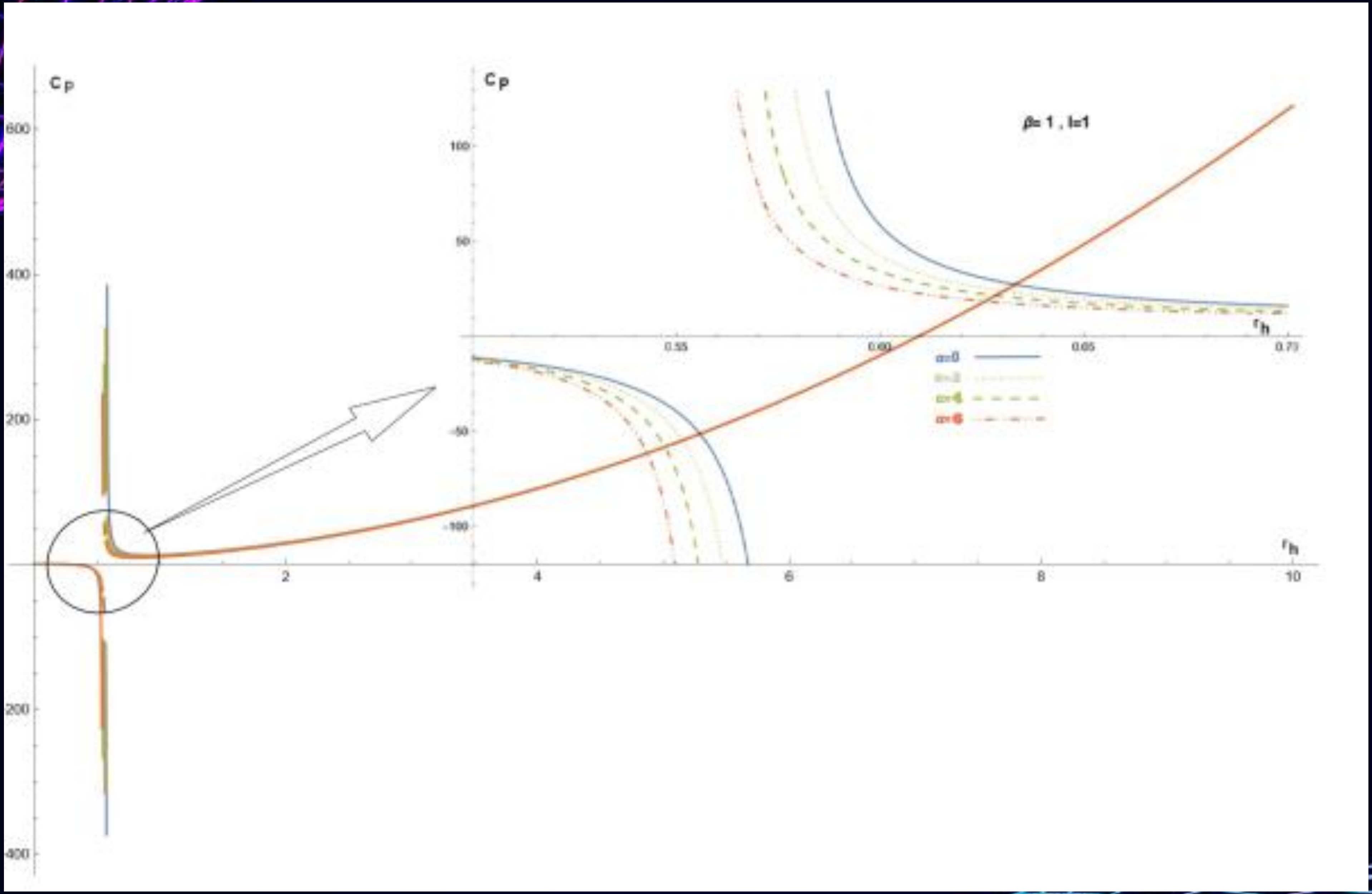


(54)

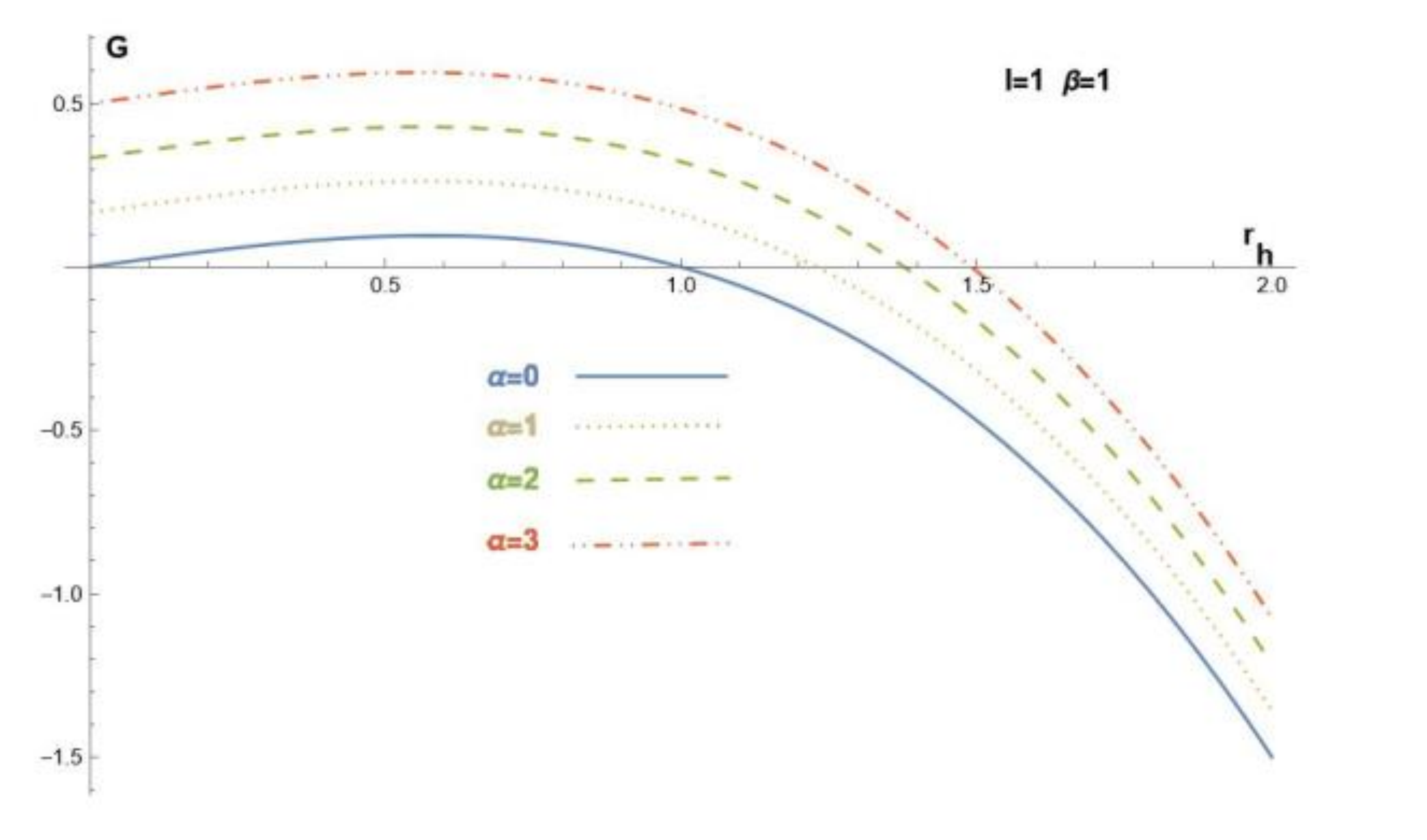
$$P = -\frac{\Lambda}{8\pi} = \frac{(d-1)(d-2)}{16\pi l^2}$$

$$r_h = 2M - \frac{8M^3}{l^2} + \mathcal{O}(M^5/l^4)$$

Discussion 3- Local thermodynamic stability



Discussion 3- Global thermodynamic stability

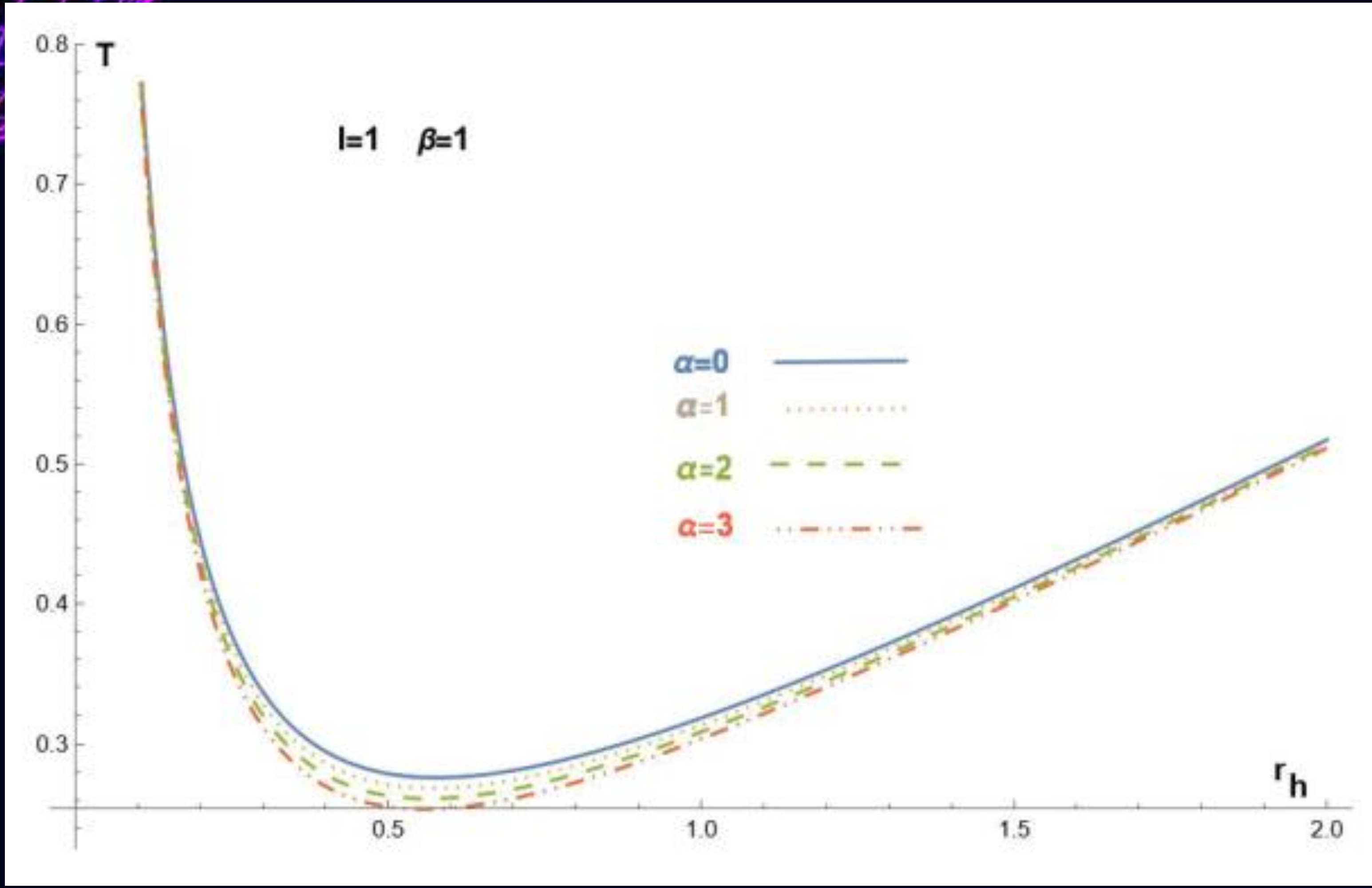


$$M \to M(r) = M + \frac{\kappa}{2} m(r)$$

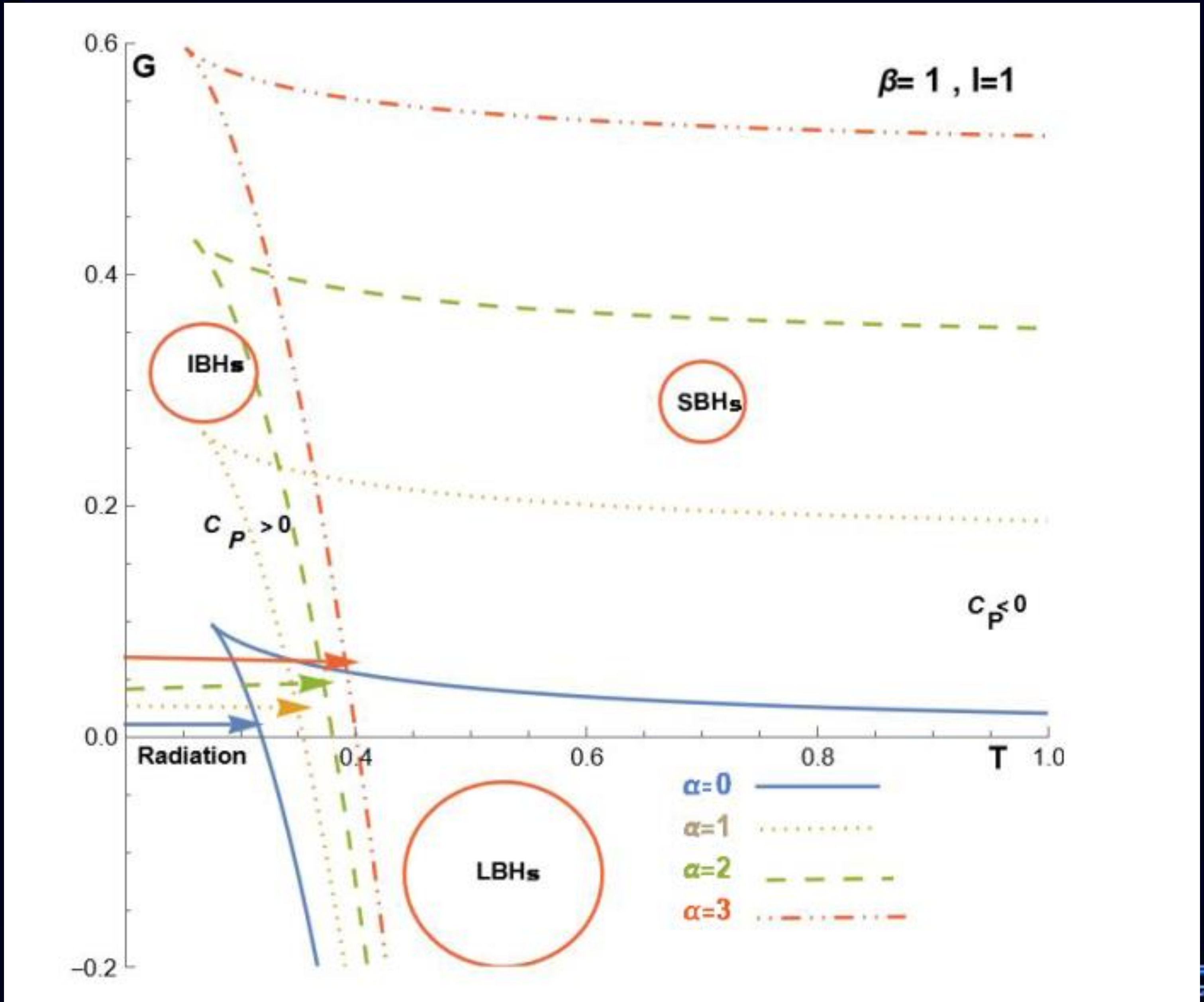
(55)

Increase in enthalpy and decrease in stability

Discussion 4- temperature behavior



Discussion 5- Hawking-Page phase transition



Summary

Deformed Ads-Schwarzschild black hole has the same thermodynamic behavior as its general relativistic counterpart. However, due to the larger mass and smaller horizon radius:

It has a lower temperature.

It has a lower local and global balance.

It has a higher Hawking-Page transition temperature.

outlook

- Use of Lovelock gravity
- Scalar Field Coupling
- Using Kerr geometry
- Holographic interpretation

Thank you for your attention