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Holography in low dimensional gravity

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Full quantum theory of gravity?

- We do not know it! (string theory, only a perturbative expansion, we do not know ST either.)
- Black holes best laboratory! (information paradox, black hole interior, ...). BH's provide **universal** features! (Entropy, Thermalization, Chaos, ...)

- The **Bekenstein-Hawking** (1970's) area law for black hole entropy is observer independent

$$S = \frac{A_{BH}}{4G_N\hbar} = \log(\text{number of microstates})$$

- Black hole is a quantum system with discrete energy levels, as viewed by a distant observer with the gap,

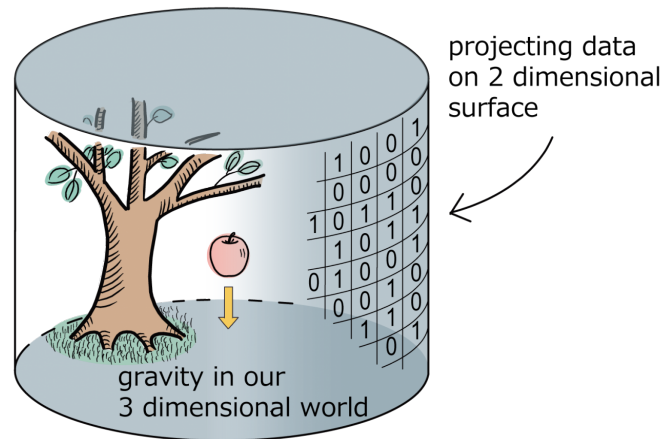
$$\delta E \sim e^{-A_{BH}/(4G_N\hbar)}, \quad \hbar \rightarrow 0.$$

- The dynamics of the Hamiltonian acting on these microstates is **chaotic**.

- **Universality** of these results suggests that a statistical description of microstates in the semi-classical limit does not need full knowledge of QG.
- Black hole area law for entropy indicates that BH's behave **holographically**.
- Holography is a property of a consistent quantum gravity ('tHooft 1993-Susskind 1994).

Holographic Principle

- A system containing gravity can be described by degrees of freedom living on its boundary rather than its bulk!



- AdS/CFT relates string theory in certain backgrounds to quantum mechanical systems and quantum field theories.
- The correspondence is conjectured to be true for all values of couplings in string theory

$$\text{quantum corct.} \sim \frac{1}{N}, \quad \text{stringy corct.} \sim \frac{1}{\lambda}.$$

- In the regime of semi-classical gravity (large N and large λ) the dual description is strongly coupled.

- Duality: In one regime of parameter space one description is easy and vice versa.
- Holography best tool to explain **non-perturbative** universal features.
- A holographic dual field theory (same dynamics) exists? Instead of answering this we ask how holography constrains the energy spectrum in quantum gravity and in particular the black holes?

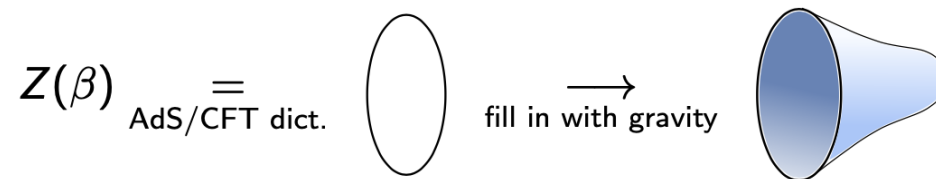
Thermal partition function

Euclidean partition function of quantum gravity with certain asymptotic condn. at a given temperature β^{-1}

$$Z(\beta) = \sum_{\text{top.}} \int Dg e^{-S_E[g]}$$

The sum is over all possible interiors including BH's,... with the same asymptotics. (uncertainty principle)

AdS/CFT dictionary, says that a given boundary cond., should be filled in by the bulk theory.



Semi-classically, in the (saddle point) approximation we evaluate the on-shell action: $Z(\beta) \approx e^{I_0(g_{cl})}$

- A dual FT with the same partition function exists?

- At the semi-classical level universal features of **black holes** such as the Bekenstien-Hawking entropy can be reproduced holgraphically.
- In low dimensions for certain toy models, **symmetry analysis** may give a Kinematic (universal) answer at the semi-classical regime.
- Toy models for **quantum gravity** in low dimensions without strings?

Quantum theory and universal features

- In interacting QFT's, universal sectors for the density of states emerge in specific limits such as high-temperature $\beta \rightarrow 0$, where the microscopic details of the theory become irrelevant and the system's behavior is dictated by symmetry principles.

- In field theories with partition function of the form

$$Z(\beta) = \text{Tr} e^{-\beta H},$$

each term in this trace (taken over the full Hilbert space of the theory) corresponds to specific eigenvalues ξ of H and counts the number of states at a given energy, resulting in the degeneracy $D(\xi)$. This degeneracy is related to the entropy after a Legendre transform

$$S(\beta) = (1 - \beta \partial_\beta) \ln Z(\beta) \rightarrow S(\xi) = \ln D(\xi)$$

- Cardy formula for CFT_2 yields the asymptotic growth of states at high temperature ($\beta \rightarrow 0$) matching the classical entropy of the BTZ black hole

$$S = 2\pi\sqrt{\frac{c}{6}L_0} + 2\pi\sqrt{\frac{c}{6}\bar{L}_0}$$

- This shows how quantities like entropy can be deduced only from Kinematics (symmetries, modular invariance, ...) without even evaluating the gravitational partition function or the gravitational action. This is the core idea of **holographic symmetries**.

Holographic symmetries

- Bondi, van der Burg, Metzner and Sachs (1962) showed that the asymptotic symmetries of 4D flat spacetime is an infinite enhancement of Poincaré symmetries; BMS symmetries. They preserve the asymptotic conditions;

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{O}(h_{\mu\nu})$$

- Brown and Henneaux (1982) showed that the asymptotic symmetry of the 3D spacetime with AdS vacuum, consists of Virasoro symmetry with a classical central charge (hint towards the AdS/CFT);

$$c = \bar{c} = \frac{3\ell}{2G}$$

- In general, is the power of these symmetries enough to even determine the effective boundary action and the whole partition function of the quantum gravity? In certain cases the answer is yes.

Jackiw-Teitelboim gravity

$$S[g, X] = \frac{\kappa}{2} \int d^2x \sqrt{g} X (R + \frac{2}{\ell^2}) + I_{\partial M}$$

- Asymptotic symmetry is the Virasoro group $Diff(S^1)$ and the boundary action is given as Goldstone theory of this group which is called Schwarzian theory

$$S = \oint \frac{d\varphi}{f'^2} \left(L_0 + \frac{c}{12} \left(\frac{f''}{f'} \right)' - \frac{1}{2} \left(\frac{f''}{f'} \right)^2 \right) .$$

- This theory coincides with the low temperature limit of a large N quantum statistical **SYK model** model.

Sachdev-Ye Kitaev model

- A statistical quantum mechanical system of N real fermions ψ^a ;

$$\psi^a(\tau)\psi^b(\tau) + \psi^b(\tau)\psi^a(\tau) = \delta^{ab}$$

- The Hamiltonian contains random fermion interactions e.g.;

$$H = -J^{abcd}\psi^a\psi^b\psi^c\psi^d, \quad \langle J_{abcd}^2 \rangle \sim J^2/N^3, \quad N \gg 1.$$

- In the strict UV ($J\tau \ll 1$) and IR ($J\tau \gg 1$) limit, the theory has reparametrization symmetry.

$$\tau \rightarrow f(\tau), \quad f(\tau) \in Diff(S^1).$$

- In the IR, this symmetry is spontaneously broken to $SL(2, \mathbb{R})$;

$$H \sim \partial_\tau, \quad D \sim \tau \partial_\tau, \quad K \sim \tau^2 \partial_\tau.$$

- Soft directions are parametrized by Goldstone modes $f(\tau)$ living in $Diff(S^1)/SL(2, \mathbb{R})$ space.

Boundary Goldstone theories

- All fields of the boundary theory live on the circle and are in some representation of $Diff(S^1)$. They are arranged as vectors, functions, 1-forms, ...

$$h(\varphi) \in Vec(S^1), \quad g(\varphi) \in C^\infty(S^1), \quad \eta(\varphi) \in \Omega^1(S^1), \dots$$

- These fields form a centrally extended symmetry group $\hat{G}(S^1)$ on the circle e.g.

1. Virasoro: $Diff(S^1)$

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}n^3\delta_{m+n,0}.$$

2. Warped Virasoro: $Diff(S^1) \ltimes C^\infty(S^1)$

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_1}{12}n^3\delta_{m+n,0}.$$

$$[L_n, P_m] = -mP_{n+m} + c_2n^2\delta_{m+n,0},$$

$$[P_n, P_m] = c_3n\delta_{m+n,0}.$$

3. BMS_3 : $\text{Diff}(S^1) \ltimes \text{Vec}(S^1)$

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_1}{12}n^3\delta_{m+n,0}.$$

$$[L_n, M_m] = (n - m)M_{n+m} + c_2n^3\delta_{m+n,0},$$

$$[M_n, M_m] = 0.$$

4. BMS_2 : $\text{Diff}(S^1) \ltimes \Omega^1(S^1)$

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c_1}{12}n^3\delta_{m+n,0}.$$

$$[L_n, Q_m] = (n + m)Q_{n+m} + (c_2n + c_3)\delta_{m+n,0}.$$

$$[Q_n, Q_m] = 0.$$

- Semi-classical holographic degrees of freedom usually have a description in terms of Goldstone modes

$$f(\varphi), g(\varphi), \dots \in \mathcal{M} = \frac{\hat{G}(S^1)}{G_0(S^1)}$$

- G_0 is the stabilizer of the vacuum solution. Vacuum has less symmetries than the Hamiltonian.
- Goldstone modes map solutions to solutions without any cost.

- These quotient subspaces are *coadjoint orbits* of $\hat{G}$ and are symplectic manifolds and can be quantized. We use the symplectic form on the coadjoint orbit as the measure of path integration which should be independent of the orbit representative.
- The partition function of this theory is 1-loop exact.

$$Z = \int \mathcal{D}f \mathcal{D}\eta e^{-S[f,\eta]} = e^{-S^{(0)}} Z_{1\text{-loop}}$$

Expand the action to 2nd order around its saddle point $f \simeq \varphi + \epsilon(\varphi)$ as $I[f, \eta] \simeq S^{(0)} + S^{(2)}[\epsilon, \eta]$.

Flat JT gravity

$$I_M[g, X] = \frac{\kappa}{2} \int d^2x \sqrt{g} \left(X \left(R + \frac{2}{\ell^2} \right) - 2\Lambda \right) + I_{\partial M}, \quad \ell \rightarrow \infty.$$

- Quantum gravity model in flat spacetime.
- Flat limit of the Schwarzian theory

Asymptotic symmetries

$$ds^2 = V(u, r) du^2 - 2 du dr, \quad V(u, r) \sim -r^2/\ell^2 + \mathcal{O}(r)$$

The vector field preserving this gauge;

$$\mathcal{L}_\xi g_{ur} = 0 = \mathcal{L}_\xi g_{rr}$$

is

$$\xi[\epsilon, \eta] = \epsilon(u) \partial_u - \left(r\epsilon'(u) + \eta(u) \right) \partial_r$$

The Lie bracket $[\xi_1, \xi_2]$ satisfy the BMS₂ algebra.

BMS₂ algebra: In the Euclidean Fourier basis

$$\ell_m = -i\xi[e^{im\varphi}, 0], \quad q_m = -i\xi[0, e^{im\varphi}], \quad \varphi = \frac{2\pi i}{\beta}\tau$$

We have the BMS₂ algebra

$$[\ell_m, \ell_n] = (m - n)\ell_{m+n}$$

$$[\ell_m, q_n] = -(m + n)q_{m+n}$$

$$[q_m, q_n] = 0$$

The elements of the BMS₂ algebra are vectors $\epsilon(\varphi)\partial_\varphi$ and one-forms $\eta(\varphi)d\varphi$ on the circle.

BMS₂ group:

The vector elements ϵ exponentiate to diffeomorphisms f on the circle $f(\varphi + 2\pi) = f(\varphi) + 2\pi$ which act on periodic one-forms η by pullback $f^*\eta = \eta \circ f$. BMS₂ group admits three non-trivial cocycles so the centrally extended algebra looks like

$$[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12}(n^3 - n)\delta_{m+n,0}$$

$$[L_n, Q_m] = -(n + m)Q_{n+m} + (bm - ia)\delta_{m+n,0}$$

$$[Q_n, Q_m] = 0$$

Phase space The fall-off conditions on the metric and the dilaton field:

$$ds^2 = -(r^2/\ell^2 + 2P(u)r + 2T(u)) du^2 - 2 du dr$$

$$X = x(u)r + y(u)$$

The variation of the bulk action

$$\delta I_M = \kappa \int_M (e.o.m.) + \kappa \int_{\partial M} (x\delta T - y\delta P)$$

We supplement a boundary term $S = I_M + I_{\partial M}$ s.t.

$$\delta I_{\partial M} = -\kappa \int_{\partial M} (x\delta T - y\delta P)$$

Boundary term

$$S = \frac{\kappa}{2} \int d^2x \sqrt{g} \left(X \left(R + \frac{2}{\ell^2} \right) - 2\Lambda \right) + I_{\partial M}$$
$$I_{\partial M} = -\kappa \oint_{\partial M} \left(Tx - Py + \frac{x'}{x}y - \Lambda \frac{y}{x} + \frac{y^2}{2\ell^2 x} \right)$$

with the following boundary condition

$$\delta \oint \frac{1}{x} = 0$$

(x, y) transform as a vector and a function. They are identified as the BMS_2 group elements:

$$x \sim \frac{1}{f'}, \quad y \sim \eta \circ f$$

Finite transformation laws

$$\tilde{T} \circ f = \frac{1}{f'^2} \left(T + \frac{c}{12} Sch(f) - b(f'\eta \circ f)' + P f'\eta \circ f \right)$$

$$\tilde{Q} \circ f = Q - b \log f' - a(f - 1)$$

The boundary action is

$$S[f, \eta] := I_{\partial M} = \oint \tilde{T} - P_0 \oint \eta + \mathcal{O}(1/\ell^2)$$

The first term is the zero mode of the stress tensor on the coadjoint orbit of BMS_2 which we call it BMS-Schwarzian term.

Summary

- The emergence of asymptotic symmetries and their role in holography for addressing universal semi-classical features.
- The role of the effective Goldstone actions to obtain quantum aspects of black holes (near horizon dynamics) using the Coadjoint orbit method. (Geometric quantization)

- In "Flat-JT" gravity, the asymptotic symmetry group is the BMS_2 group which is a semi-direct product of the Virasoro group acting on one-forms on S^1 .
- The structure of the group already furnishes the phase space of the theory which enables us to evaluate the thermal partition function.
- Especially it is tempting if there exist an SYK-like model realizing this symmetry in some regime.

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