Duality and effective actions

by

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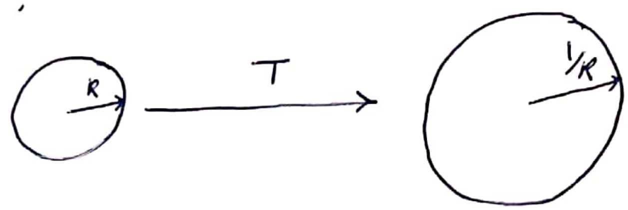
T-duality of Heterotic theory:

$$S_H^{(0)} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G}\, e^{-2\Phi}\left[R + 4\nabla_\alpha\Phi\nabla^\alpha\Phi - \frac{1}{12}H^2 - \frac{1}{4}\mathrm{Tr}(F^2)\right]$$

↳ has exact genus $e^{-2\Phi}$.

- reduce on circle:



$\bar{\Phi} \xrightarrow{T} \bar{\Phi}, \ldots$ $\quad e^{\bar{\Phi}}$ ↝ coupling const. in 9-dim theory

weak-weak duality

$\mathbb{Z}_2$-group

$S_H\big|_{\text{circle}} \xrightarrow{T} S_H\big|_{\text{circle}}$ ↳ it fixes action.

- at 2-derivative level: $\quad S_H^{(0)} \xrightarrow{T} S_H^{(0)}$ ✓

- at 4-derivative level:

$$H \xrightarrow{G.S.} H + \frac{3}{2}\alpha'\Omega$$

$\alpha' H\Omega$ ↳ is not invariant

if $\bar{\Phi} \xrightarrow{T} \bar{\Phi} + \alpha'\Delta\bar{\Phi}, \ldots$ ↝ $\mathbb{Z}_2$-group

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$\alpha' H\Omega + \alpha' S_H^{(1)} \xrightarrow{T} \alpha' H\Omega + \alpha' S_H^{(1)}$ ⟶ $S_H^{(1)}$ can be found.

- at 6-derivative level:

$$\alpha'^2 \Omega^2 + \alpha'^2 S^{(1)}_{H.}\Big|_{H+\frac{3}{2}\alpha'\Omega} \rightsquigarrow \text{is not invariant.}$$

$$\alpha'^2 \Omega^2 + \alpha'^2 S^{(1)}_{H.}\Big|_{H+\frac{3}{2}\alpha'\Omega} + \alpha'^2 S^{(2)}_{H.} \xrightarrow{T} \alpha'^2 \Omega^2 + \alpha'^2 S^{(1)}_{H.}\Big|_{H+\frac{3}{2}\alpha'\Omega} + \alpha'^2 S^{(2)}_{H.}$$

$\hookrightarrow$ $S^{(2)}_{H.}$ can be found $\rightsquigarrow$ arXiv: 2406.02960

- Similarly can be found for higher-derivative terms:

$$S^{(0)}_{H.} + \alpha' S^{(1)}_{H.} + \alpha'^2 S^{(2)}_{H.} + \cdots$$

$\to$ can be found by T-duality

$\to$ must be supersymmetric as well.

$\to$ all have exact genus $e^{-2\phi}$

- Impose T-duality in most general higher-derivative terms, one finds also

$$\underbrace{\zeta(3)\left[\alpha'^3 S^{(3)}_{\zeta(3)} + \alpha'^4 S^{(4)}_{\zeta(3)} + \cdots\right] + \zeta(5)\left[\alpha'^4 S^{(4)}_{\zeta(5)} + \alpha'^5 S^{(5)}_{\zeta(5)} + \cdots\right] + \cdots}$$

all have $e^{-2\phi}$, but have quantum corrections, because they are not connected to $S^{(0)}_{H_0}$.

- can the higher genus corrections be found by T-duality?

T-duality of Type I theory;

$$S_{I}\Big|_{\text{circle}} \xrightarrow{\ T\ } S_{IIA}\Big|_{\text{circle}/\mathbb{Z}_2}$$

$\bar{\Phi} \longrightarrow \bar{\Phi}$, ... weak-weak duality

RR field

$$S_{I}^{(0)} = -\frac{2}{k^2}\int d^{10}x \sqrt{-G}\left[e^{-2\phi}(R + 4\nabla_\alpha\phi\nabla^\alpha\phi) - \frac{1}{2}|F^{(3)}|^2\right]$$

$$S_{IIA}^{(0)} = -\frac{2}{k^2}\int d^{10}x \sqrt{-G}\left[e^{-2\phi}(R + 4\nabla_\alpha\phi\nabla^\alpha\phi - \frac{1}{12}H^2) - \frac{1}{2}|F^{(2)}|^2 - \frac{1}{2}|F^{(4)}|^2\right]$$

- at 2-derivative :

$$S_{I}^{(0)}\Big|_{\text{circle}} \xrightarrow{\ T\ } S_{IIA}^{(0)}\Big|_{\text{circle}/\mathbb{Z}_2} \quad \checkmark$$

- at 4-derivative :

$$F^{(3)} \xrightarrow{\ G.S.\ } F^{(3)} + \frac{3}{2}\alpha'\Omega \quad \rightsquigarrow \text{Type I}$$

$$H \longrightarrow H \quad \rightsquigarrow \text{Type IIA (non-chiral theory)}$$

$\alpha' F^{(3)}\Omega$ $\rightsquigarrow$ does not transforms to any term in IIA

- rescale RR; $F^{(3)} = e^{-\phi}\tilde{F}^{(3)}$ $\Longrightarrow |F^{(3)}|^2 = e^{-2\phi}|\tilde{F}^{(3)}|^2$ $\rightsquigarrow$ sphere-level (classic)

$\alpha' F^{(3)}\Omega = \alpha' e^{-\phi}\tilde{F}^{(3)}\Omega$ $\rightsquigarrow$ disk-level (quantum)

$\rightarrow$ T-duality exist <u>only</u> at classical level.

S-duality of Heterotic/Type I :

$$S^{(0)}_{H.} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G'}\,e^{-2\phi'}\left[R' + 4\nabla_\alpha\phi'\nabla^\alpha\phi' - \frac{1}{12}H'^2 - \frac{1}{4}Tr(F'^2)\right]$$

$$S^{(0)}_{I.} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G}\left[e^{-2\phi}(R + 4\nabla_\alpha\phi\nabla^\alpha\phi) - \frac{1}{12}H^2 - \frac{e^{-\phi}}{4}Tr(F^2)\right]$$

$\hookrightarrow$ KR, $F^{(3)}\cdot F^{(3)}$

have exact genus (classic)

- it has the following duality:

$$\phi' = -\phi\ ,\quad G'_{\alpha\beta} = e^{-\phi}G_{\alpha\beta}\ ,\quad B'_{\alpha\beta} = B_{\alpha\beta}\ ,\quad A' = A$$

e^{ϕ} coupling in 10-dim $\rightsquigarrow$ weak-strong duality

- at two-derivative level: $S^{(0)}_{H} \xrightarrow{S} S^{(0)}_{I.}$ ✓

- at 4-derivative level:
  " 6-derivative "

$$H' \xrightarrow{G.S.} H' + \frac{3}{2}\alpha'\Omega'$$

$$H \xrightarrow{G.S.} H + \frac{3}{2}\alpha'\Omega$$

if $\Omega' = \Omega$ $\Rightarrow$ $S_{H.} \xrightarrow{S} S_{I.}$ $\rightsquigarrow$ at $\underbrace{\text{4 \& 6 derivatives}}_{\text{from G.S.}}$ as well.

- How about other 4-& 6-derivative terms?

- Weak-expansion at Heterotic and Type I theories:

$$S_{H.}' = \sum_{g=0}^{\infty} \int d x'^{10}\, e^{2(g-1)\phi'} \sqrt{-G'}\, \mathcal{L}_g(\alpha') \quad ; \quad e^{\phi'} < 1$$

$$S_{I.} = \sum_{g,b,c=0}^{\infty} \int d x^{10}\, e^{(2g+b+c-2)\phi} \sqrt{-G}\, \mathcal{L}_{(g,b,c)}(\alpha') \quad ; \quad e^{\phi} < 1$$

$$\underbrace{e^{\phi'} < 1}_{\text{Heterotic theory}} \xrightarrow{S} \underbrace{e^{-\phi} > 1}_{\text{Type I theory}}$$

→ can not be compared with the above weak-expansion at Type I.

- However, consider a subclass of these expansions:

$$S_{H.}^{exact} = \sum_{g=0}^{\infty} \int d x'^{10}\, e^{2(g-1)\phi'} \sqrt{-G'}\, \mathcal{L}_g^{exact}(\alpha') \quad \text{→ at any } e^{\phi'}$$

$$S_{I.}^{exact} = \sum_{g,b,c=0}^{\infty} \int d x^{10}\, e^{(2g+b+c-2)\phi} \sqrt{-G}\, \mathcal{L}_{(g,b,c)}^{exact}(\alpha') \quad \text{→ at any } e^{\phi}$$

$$S_H^{exact} \xrightarrow{S} S_I^{exact}$$

- classical $\underbrace{S_{H.}^{(0)} + \alpha' S_{H.}^{(1)} + \alpha'^2 S_{H.}^{(2)} + \cdots}_{\substack{\text{- exact } e^{-2\phi} \\ \text{- can be found by T-duality}}} \xrightarrow{S} \underbrace{S_I^{(0)} + \alpha' S_I^{(1)} + \alpha'^2 S_I^{(2)} + \cdots}_{e^{-2\phi},\, e^{-\phi},\, e^{0},\, e^{\phi}, \cdots}$

- one-loop: $B'R'^4 + \cdots \xrightarrow{S} BR^4 + \cdots$ → are not interested.

↳ connected by supersymmetry

- To explicitly find higher-derivative exact couplings in Type I:

1- Higher-derivative corrections to S-duality.

2- Relations between field redefinitions in two theories.

- Is there higher-derivative correction to S-duality?

$$\phi \xrightarrow{S} -\phi \xrightarrow{S} \phi$$
$$G \xrightarrow{S} e^{-\phi}G \xrightarrow{S} e^{\phi}e^{-\phi}G = G \quad \Longrightarrow \quad Z_2\text{-group.}$$
$$B \xrightarrow{S} B \xrightarrow{S} B$$

their higher-derivative correct must satisfy Z_2-group

- For example:

$$\phi' = -\phi + \alpha' \Delta\phi(\phi, G, B)$$

$$\underbrace{(\phi')'}_{\phi} = -\phi' + \alpha'\Delta\phi(\phi', G', B') = -\left[-\phi + \alpha'\Delta\phi(\phi, G, B)\right] + \alpha'\Delta\phi(-\phi, e^{-\phi}G, B)$$

$$\Longrightarrow \Delta\phi(\phi, G, B) = \Delta\phi(-\phi, e^{-\phi}G, B)$$

if e.g. $\Delta\phi = e^{\phi/2} G^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi$ $\rightsquigarrow$ it satisfies the Z_2-condition.

$$S_{H.} \xrightarrow{\phi' = -\phi + \alpha'\Delta\phi} S_I \quad \Longrightarrow \quad e^{-2\phi'} \longrightarrow \alpha' \underbrace{e^{2\phi}\Delta\phi}_{e^{2\phi + \phi/2} = e^{\frac{5}{2}\phi}}$$

$\rightsquigarrow$ is not allowed

$$\Longrightarrow \Delta\phi = 0, \quad \Delta G = 0, \quad \Delta B = 0$$

- Relation between higher-derivative field redefinitions:
- in field redefinition, one is allowed to remove any term by using e.o.m.

e.o.m. in type I :

$$\begin{cases} D\phi \equiv -2\, e^{-2\phi}\left(R+4\nabla_\alpha \nabla^\alpha\phi - 4\nabla_\alpha\phi\nabla^\alpha\phi\right)=0 \\ DB^{\alpha\beta} \equiv \frac{1}{2}\nabla_\gamma H^{\alpha\beta\gamma}=0 \\ DG^{\alpha\beta} \equiv \frac{1}{4}(H^2)^{\alpha\beta} - \left(\frac{1}{4}D\phi + \frac{1}{24}H^2\right)G^{\alpha\beta} - e^{-2\phi}\left(R^{\alpha\beta}+2\nabla^\alpha\nabla^\beta\phi\right)=0 \end{cases}$$

e.o.m. in Heterotic theory:

$$\begin{cases} D\phi' \equiv -2\, e^{-2\phi'}\left(R'+4\nabla_\alpha \nabla^\alpha\phi' - 4\nabla_\alpha\phi'\nabla^\alpha\phi' - \frac{1}{12}H'^2\right)=0 \\ DB'^{\alpha\beta} \equiv \frac{1}{2}\nabla_\gamma\left(e^{-2\phi'} H'^{\alpha\beta\gamma}\right)=0 \\ DG'^{\alpha\beta} \equiv \frac{1}{4}e^{-2\phi'}(H'^2)^{\alpha\beta} - \frac{1}{4}D\phi' G'^{\alpha\beta} - e^{-2\phi'}\left(R'^{\alpha\beta}+2\nabla^\alpha\nabla^\beta\phi'\right)=0 \end{cases}$$

- consider for example the following coupling in Type I :

$$e^{\phi}H^2\left(\frac{1}{12}e^{2\phi}H^2 + \frac{1}{2}R + \nabla_\alpha\phi\nabla^\alpha\phi\right) \rightsquigarrow \text{can not be removed by field redefinition}$$

under S-duality

$$\xrightarrow{S} e^{-2\phi'}H'^2\left(\frac{1}{12}H'^2 + \frac{1}{2}R' + \frac{7}{2}\nabla_\alpha\nabla^\alpha\phi' - 5\nabla_\alpha\phi'\nabla^\alpha\phi'\right)$$

can be written as

$$\left(-30R' - 135\nabla_\alpha\nabla^\alpha\phi' + 150\nabla_\alpha\phi'\nabla^\alpha\phi'\right)D\phi' + \left(\frac{1}{2}H'^2 - 15R' - 60\nabla_\alpha\nabla^\alpha\phi' + 60\nabla_\alpha\phi'\nabla^\alpha\phi'\right)DG'^{\mu}_{\mu}$$

⇒ field redefinition in type I depends on scheme at couplings in heterotic theory.

$$S_{H.}^{(1)} = -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G'}\,e^{-2\phi'}\Big[2H'\Omega' + \frac{1}{4}\mathrm{Tr}(F_\alpha F^{\gamma\alpha\beta})\mathrm{Tr}(F_\beta{}^{\delta}F_{\gamma\delta}) + \cdots\Big]$$

↳ minimal scheme.

↳ for $F=0$ it reduces to M.T. action.

↳ 9 other terms

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if $S_{H.}^{(1)} \xrightarrow{S} S_{I.}^{(1)}$ ↳ it contains many terms that should be removed by field redefinitions in type I.

- However, we have to use only allowed f.r.

- to use allowed f.r. we use the following steps:

1- $S_I^{(1)}$ ↳ maximal basis; has 42 terms; $S_I^{(1)} = -\frac{2}{\kappa^2}\int d^{10}x\sqrt{-G}\,[a_1 R^2 + a_2 H^4 + \cdots]$

↳ arXiv: 2404.00957

2- $S_I^{(1)} \xrightarrow{S} \tilde{S}_H^{(1)}$ ↳ in terms of 42 parameters $a_1, a_2, \dots, a_{42}$

3- $S_H^{(1)} = \tilde{S}_H^{(1)}$, up to total derivative terms, field redefinition, and Bianchi Iden.

↳ it produces 24 relations.

4- replace these relations into $S_I^{(1)}$ ↳ contains 18 arbitrary parameters

↳ any choice for them gives a particular schem in type I.

- the scheme which has no $\nabla_\alpha \phi$; $\rightsquigarrow$ 3 parameters still remain
- a specific choice for them gives the following action in type I.

$$S_I^{(1)} = -\frac{2\alpha'}{8\kappa^2}\int d^{10}x\sqrt{-G}\Big\{ e^{\phi}\Big(\frac{1}{8}\mathrm{Tr}(F_\alpha{}^{\gamma}F^{\alpha\beta})\mathrm{Tr}(F_\beta{}^{\delta}F_{\gamma\delta}) + \cdots\Big) + \Big(\frac{1}{4}\mathrm{Tr}(F^{\alpha\beta}F^{\gamma\delta})\tilde{H}_{\alpha\gamma}{}^{\epsilon}\tilde{H}_{\beta\delta\epsilon} + \cdots\Big) + e^{-\phi}\Big(\frac{3}{64}\mathrm{Tr}(F_{\alpha\beta}F^{\alpha\beta})R + \cdots + 2\tilde{H}^{\alpha\beta\gamma}\Omega_{\alpha\beta\gamma}\Big)\Big\}$$

- none of $e^{\phi}(\cdots)$, $(\cdots)$, $e^{-\phi}(\cdots)$ is sphere-level
- there is no such couplings in type IIA theory
- this is another evidence that T-duality is only at classical level.
- NS-NS couplings appear at disk-level.

S-duality of 6-dim Heterotic/Type IIA : (only NS-NS) (only α'-order) ⑩

Hete. on $T^4 \sim$ IIA on K3 ;

$$\phi' = -\phi \ , \quad G'_{\mu\nu} = e^{-2\phi} G_{\mu\nu} \ , \quad H' = e^{-2\phi} * H$$

- consider for example :

$$\underbrace{e^{-2\phi'}}_{\text{sphere-level}} \underbrace{\sqrt{-G'}\, R'_{\mu\nu\alpha\beta} R'^{\mu\nu\alpha\beta}}_{H.\ \text{on}\ T^4} \xrightarrow{\ S\ } \underbrace{\sqrt{-G}\, R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}}_{\text{IIA on K3 at torus-level}} + \underbrace{\cdots}_{\text{involve dilaton}}$$

- to remove dilaton terms, one should use field redefinition that can be done as in H./I. case.

- S-duality can be used inversely;

$$\underbrace{\text{torus-level in } H.\ \text{on } T^4}_{\substack{0 \text{ at order } \alpha' \\ \text{because sphere-level at order } \alpha' \text{ is exact.}}} \xrightarrow{\ S\ } \underbrace{\text{sphere-level in IIA on K3}}_{\substack{\text{must be } 0 \\ \text{at order } \alpha'}}$$

- in K3-manifold: $ds^2 = G_{\mu\nu}\, dx^\mu dx^\nu + g_{ab}\, dy^a dy^b$

$$\int_{K_3} dy^4 \sqrt{g} = V \quad \& \quad \frac{1}{32\pi^2}\int_{K3} dy^4 \sqrt{g}\, R_{abcd} R^{abcd} = 24$$

$\rightarrow$ 8-derivative couplings at IIA $\xrightarrow{\text{K3-reduction}}$ 4-derivative + 8-derivative

$$S^{(3)}_{IIA} = \frac{2\alpha'^3 \zeta(3)}{2^6 \kappa^2} \int dx^{10} \sqrt{-G}\, e^{-2\phi} \Big[R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \Big(-\frac{5}{32} (H^2)^{\delta\gamma} (H^2)_{\gamma\delta}$$

$$+ \frac{5}{4} H_\alpha{}^{\delta\varepsilon} H R^{\alpha\beta\gamma}{}_{\beta\delta\gamma\varepsilon} - \frac{5}{24} \nabla_\mu H_{\alpha\beta\gamma} \nabla^\mu H^{\alpha\beta\gamma} \Big) + \cdots \Big]$$

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K3-reduction produce the following 4-derivative couplings:

$$S^{(1)}_{IIA} = \frac{48\pi^2 \alpha'^3 \zeta(3)}{\kappa^2} \int dx^6 \sqrt{-G} \left(-\frac{5}{32} (H^2)^{\delta\gamma} (H^2)_{\gamma\delta} + \frac{5}{4} HHR - \frac{5}{24} \nabla H \nabla H \right)$$

-up to some total derivative term it can be written as

$$c\Big[DB^{\alpha\beta} \Big(\frac{5}{4} \nabla_\gamma H_{\alpha\beta}{}^\gamma \Big) + D\phi \Big(\frac{5}{32} H^2 + \frac{5}{2} \nabla_\alpha \phi \nabla^\alpha \phi \Big) + DG^{\alpha\beta} \Big(\frac{5}{8} (H^2)_{\alpha\beta} + 5 \nabla_\alpha \nabla_\beta \phi \Big) \Big]$$

$$c = -\frac{24\pi^2 \zeta(3) \alpha'^2}{V}$$

it confirms the NS-NS couplings at order α'^3 that are found by T-duality.

- A proposal for quantum gravity:

- Heterotic string theory has the following action:

$$S = \sum_{g=0}^{\infty} \int d x^{10} \, e^{2(g-1)\phi} \sqrt{-G} \, \mathcal{L}_g(\alpha')$$

$\hookrightarrow$ very difficult to find it specially for higher-genus.

- if S is known, then S-matrix elements can be calculated; <u>UV finite</u>.

- on the other hand

known; point-like

$$S_{sugra} = \int d x^{10} \sqrt{-G} \, e^{-2\phi} \mathcal{L}_0(0) \subset S$$

$\downarrow$ the corresponding S-matrix elements are not UV finite.

- impose the stringy symmetry of T-duality on it

$$S_{exact} = \int d x^{10} \sqrt{-G} \, e^{-2\phi} \mathcal{L}_{exact}(\alpha')$$

$\to$ stringy-like

$\to$ can be found by T-duality.

$\to$ has no quantum correction.

$\downarrow$ Is the corresponding S-matrix elements are finite?

- If so, it can be considered as a quantum gravity which is much simpler than the usual string theory.

- Is there a special chiral string that produces the above spacetime action.

Thank you