Calculation Of D-Brane Action by T-Duality at α′ Order In Bosonic String Theory

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Content

Introduction

Effective Action :

- string theory —low energy→ Super Gravity
- massive field → Higher Order Derivative

D-Brane :

- Probe (Don’t Change Background)
- Geometry Space time
  - World Volume : (along) (p) (a,b,…)
  - Transvers : (orthogonal)(D-p) (i,j,….)

T-Duality :

- R ↔ (α′ / R)
- Action Invariant
  - Bulk : $S \xrightarrow{T} S$
  - D-Brane : $S_{D_p} \xrightarrow{T} S_{D_{p\pm1}}$

Theory Bosonic (D=26)

Independent Couplings

All Contraction : (Close Strings) : $(R,\Omega,\tilde{\nabla}\Omega,\tilde{B},\tilde{\nabla}\tilde{B},\tilde{\nabla}\tilde{\nabla}\tilde{B},H,\nabla H,\nabla\phi,\nabla\nabla\phi)$

Action D-Brane (α' order) : $S'_p = -\frac{\alpha' T_p}{2}\int d^{p+1}\sigma\sqrt{\det\tilde{G}_{ab}}\,L'(R,H,\nabla\phi,\Omega,\tilde{B},\tilde{\nabla}\tilde{B},\tilde{\nabla}\tilde{\nabla}\tilde{B})$

Constraint on independent Coupling

Parity : (number even of B-Field)

Field Redefinition : (only Bulk Fields)

Impose Equation of Motion : (Don't Consider)
- $\nabla_\mu H^{\mu\alpha\beta}$
- $\nabla^\mu\nabla_\mu\phi$
- $R_{\alpha\beta}$ (Ricci Tensor)
- R (Ricci Scalar)

Bianchi Identity
- $R_{\mu[\alpha\beta\nu]} = 0 \longrightarrow$ Local frame
- $\tilde{\nabla}_{[a}\tilde{B}_{cd]} = 0 \longrightarrow \tilde{B}_{ab} = \tilde{\nabla}_a A_b - \tilde{\nabla}_b A_a$

: $S' \to S$ Not Gauge inv and Not Covariant

Independent Couplings

- Total Derivative : $J = -\frac{\alpha' T_p}{2}\int d^{p+1}\sigma e^{-\phi}\tilde{\nabla}_{\alpha}(J^{\alpha})$

  Vector (J^{α}) : All contraction ($\tilde{\nabla}\tilde{B}, \tilde{\nabla}\tilde{\nabla}\tilde{B}$)

- $S - S' + J = 0$: solve : number relation : number Couplings → parity, Total derivative, Bianchi Identity

- Arrange independent Couplings : number of B-Field (L_m) → m=0,1,2,3,4

m=0 → 11

m=1 → 6

m=2 → 35

m=3 → 17

m=4 → 7

Independent Couplings

m=0 :

$$L_0 = b_{11} H_{abc} H^{abc} + b_4 H_{ab\mu} H^{ab\mu} + b_2 H_{a\mu\nu} H^{a\mu\nu} + b_1 H_{\mu\nu\rho} H^{\mu\nu\rho} + b_5 R_{ab}{}^{ab}$$
$$+ b_{45} \nabla_\mu \phi \nabla^\mu \phi + b_{42} \Omega_\mu{}^a{}_a \nabla^\mu \phi + b_{47} \nabla_a \phi \nabla^a \phi + a_1 \Omega_\mu{}^b{}_b \Omega^{\mu a}{}_a + a_2 \Omega_{\mu ab} \Omega^{\mu ab} + a_8 \tilde{\nabla}_a \tilde{B}_{bc} \tilde{\nabla}^a \tilde{B}^{bc}$$

m=1 :

$$L_1 = b_{27} \tilde{B}^{ab} H_{bc\mu} \Omega^\mu{}_a{}^c + b_{28} \tilde{B}^{ab} H_{ab\mu} \Omega^{\mu c}{}_c + b_{36} \tilde{B}^{bc} H_{abc} \nabla^a \phi$$
$$- b_{50} \tilde{B}^{bc} \nabla^a \phi \tilde{\nabla}_b \tilde{B}_{ac} + b_{51} \tilde{B}_a{}^b \nabla^a \phi \tilde{\nabla}^c \tilde{B}_{bc} + b_{35} \tilde{B}^{ab} H_{ab\mu} \nabla^\mu \phi$$

m=2 : $L_3 = b_{29} B_a^c B^{ab} B^{de} H_{de\mu} \Omega^\mu_{bc} + ...$

m=3 :

$$L_3 = b_{29} \tilde{B}_a^c \tilde{B}^{ab} \tilde{B}^{de} H_{\mu de} \Omega^\mu{}_{bc} + b_{30} \tilde{B}_a^c \tilde{B}^{ab} \tilde{B}^{de} H_{ce\mu} \Omega^\mu{}_{bd} + b_{31} \tilde{B}_a^c \tilde{B}^{ab} \tilde{B}_b{}^d H_{de\mu} \Omega^\mu{}_c{}^e$$
$$+ b_{32} \tilde{B}_{ab} \tilde{B}^{ab} \tilde{B}^{cd} H_{de\mu} \Omega^\mu{}_c{}^e + b_{33} \tilde{B}_a^c \tilde{B}^{ab} \tilde{B}_b{}^d H_{cd\mu} \Omega^{\mu e}{}_e + b_{34} \tilde{B}_{ab} \tilde{B}^{ab} \tilde{B}^{cd} H_{cd\mu} \Omega^\mu{}_e{}^e$$
$$+ b_{39} \tilde{B}_b{}^d \tilde{B}^{bc} \tilde{B}_c{}^e H_{ade} \nabla^a \phi + b_{40} \tilde{B}_{bc} \tilde{B}^{bc} \tilde{B}^{de} H_{ade} \nabla^a \phi + b_{41} \tilde{B}_a{}^b \tilde{B}^{de} \tilde{B}_b{}^c \nabla^a \phi$$
$$- b_{56} \tilde{B}_b{}^d \tilde{B}^{bc} \tilde{B}_c{}^e \tilde{\nabla}_d \tilde{B}_{ae} \nabla^a \phi - b_{58} \tilde{B}_{bc} \tilde{B}^{bc} \tilde{B}^{de} \tilde{\nabla}_d \tilde{B}_{ae} \nabla^a \phi + b_{57} \tilde{B}_a{}^b \tilde{B}^{cd} \tilde{B}_c{}^e \tilde{\nabla}_d \tilde{B}_{be} \nabla^a \phi$$
$$- b_{59} \tilde{B}_a{}^b \tilde{B}^{de} \tilde{B}_b{}^c \tilde{\nabla}_d \tilde{B}_{ce} \nabla^a \phi + b_{60} \tilde{B}_a{}^b \tilde{B}_{dc} \tilde{B}^{dc} \tilde{\nabla}^e \tilde{B}_{be} \nabla^a \phi + b_{61} \tilde{B}_a{}^b \tilde{B}_b{}^c \tilde{B}_c{}^d \tilde{\nabla}^e \tilde{B}_{de} \nabla^a \phi$$
$$+ b_{37} \tilde{B}_a{}^c \tilde{B}_b{}^d \tilde{B}^{ab} H_{cde} \nabla^\mu \phi + b_{38} \tilde{B}_{ab} \tilde{B}^{ab} \tilde{B}^{cd} H_{cd\mu} \nabla^\mu \phi$$

Reduction

- After Reduction (Base Space) :
  - reduction Action : $S = S^0 + \alpha'(S_1^1 + S_2^1 + S_3^1 + ...) + (\alpha')^2 ....$
  - reduction Fields : ($B, G, \phi, \tilde{G}$)

$$M^{(26)} = M^{(25)} \times S^{(1)}$$

- Constraint on Base Space
  - Number (m): m > 0
  - Reduction Direction :
    - WV : Diagonal Base Space Metric
    - Tra : → $g_{\tilde{a}} = 0$, $\partial g_{\tilde{a}} \neq 0$

. Second Fundamental form : ($\Omega^{\mu}{}_{ab} = \partial_a \partial_b X^{\mu} + \partial_a X^{\alpha} \partial_b X^{\beta} \Gamma_{\alpha\beta}{}^{\mu}$)

- Static Gauge
  - $x^a = \sigma^a$
  - $x^i = 0$

$\Omega^{\mu}{}_{ab}$ —in general→ { $\Omega^{c}{}_{ab} = 0$, $\Omega^{i}_{ab} = \Gamma^{i}_{ab}$ } in static gauge —if $\Omega^{c}{}_{ab} = 0$→ $\tilde{G}^{ab} G_{b\mu} = \delta^{a}{}_{\mu}$

- WV →
  - $\bar{g}^{\tilde{\mu}\tilde{\nu}} = \begin{pmatrix} \bar{g}_{\tilde{a}\tilde{b}} & 0 \\ 0 & \bar{g}_{\tilde{i}\tilde{j}} \end{pmatrix}$
  - No Constraint On $g_{\tilde{a}}$
- Tra →
  - $g_{\tilde{a}} = 0$
  - $\partial g_{\tilde{a}} \neq 0$

Reduction Rules (Fields)

- Pull Back Metric : $\tilde{G}_{ab} = \frac{\partial X^{\mu}}{\partial \sigma^{a}} \frac{\partial X^{\nu}}{\partial \sigma^{b}} G_{\mu\nu}$

(W V) reduction → $\tilde{G}_{ab} = \begin{pmatrix} \bar{g}_{\tilde{a}\tilde{b}} + e^{\varphi} g_{\tilde{a}} g_{\tilde{b}} & e^{\varphi} g_{\tilde{a}} \\ e^{\varphi} g_{\tilde{b}} & e^{\varphi} \end{pmatrix}$ inverse → $\tilde{G}^{ab} = \begin{pmatrix} \bar{g}^{\tilde{a}\tilde{b}} & -g^{\tilde{a}} \\ -g^{\tilde{b}} & e^{-\varphi} + g^{\tilde{c}} g_{\tilde{c}} \end{pmatrix}$

(Tra) reduction → $\tilde{G}_{\tilde{a}\tilde{b}} = \bar{g}_{\tilde{a}\tilde{b}} + e^{\varphi} g_{\tilde{a}} g_{\tilde{b}}$ inverse → $\tilde{G}^{\tilde{a}\tilde{b}} = \bar{g}^{\tilde{a}\tilde{b}} - \frac{g^{\tilde{a}} g^{\tilde{b}}}{e^{-\varphi} + g^{\tilde{c}} g_{\tilde{c}}}$

Metric → $G^{\mu\nu} = \begin{pmatrix} g^{\tilde{\mu}\tilde{\nu}} & -g^{\tilde{\mu}} \\ -g^{\tilde{\nu}} & e^{-\varphi} + g^{\tilde{\alpha}} g_{\tilde{\alpha}} \end{pmatrix}$, $G_{\mu\nu} = \begin{pmatrix} g_{\tilde{\mu}\tilde{\nu}} + e^{\varphi} g_{\tilde{\mu}} g_{\tilde{\nu}} & e^{\varphi} g_{\tilde{\mu}} \\ e^{\varphi} g_{\tilde{\nu}} & e^{\varphi} \end{pmatrix}$

B-Field → $B_{\mu\nu} = \begin{pmatrix} b_{\tilde{\mu}\tilde{\nu}} + \frac{1}{2} b_{\tilde{\mu}} g_{\tilde{\nu}} - \frac{1}{2} b_{\tilde{\nu}} g_{\tilde{\mu}} & b_{\tilde{\mu}} \\ -b_{\tilde{\nu}} & 0 \end{pmatrix}$

Dilaton → $\phi = \bar{\phi} + \frac{\varphi}{4}$

Reduction of Action

Expand D-Brane Action(Low Energy)

- $S = S^0 + \alpha'(S_1^1 + S_2^1 + S_3^1 + \ldots.) + (\alpha')^2 \ldots.$
- DBI Action (S^0) :

$$S_p^0 = -T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(G_{ab} + B_{ab} + 2\pi\alpha' F)} \xrightarrow[\text{Close Strings}]{\text{Static Gauge}} S_p^0 = -T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{-\det(G_{ab} + B_{ab})}$$

Reduction of DBI action :

WV : $S_{p-1}^{0w} = -T_{p-1} \int d^p \sigma e^{-\bar{\phi} + \varphi/4} \sqrt{-\det(\tilde{g}_{\tilde{a}\tilde{b}} + \tilde{b}_{\tilde{a}\tilde{b}} + g_{[\tilde{a}} b_{\tilde{b}]} + e^{-\varphi} b_{\tilde{a}} b_{\tilde{b}})}$

Tra : $S_{p-1}^{0t} = -T_{p-1} \int d^p \sigma e^{-\bar{\phi} - \varphi/4} \sqrt{-\det(\tilde{g}_{\tilde{a}\tilde{b}} + \tilde{b}_{\tilde{a}\tilde{b}} + b_{[\tilde{a}} g_{\tilde{b}]} + e^{-\varphi} g_{\tilde{a}} g_{\tilde{b}})}$

Not gauge inv ($U(1) \times U(1)$)

Momentum Vector : $g_{\tilde{a}}$

Winding Vector : $b_{\tilde{a}}$

T Duality of (α′°)DBI Action

T-Duality:

- (Without Correction) : $S_p^{0w} \xrightarrow[\varphi \to -\varphi,\, b \leftrightarrow g]{\text{Buscher Rules}} S_{p-1}^{0wt} \longrightarrow S_{p-1}^{0wt} - S_{p-1}^{0t} = 0$

- (With Correction) : $S_p^{0w} \xrightarrow[g_{\tilde{a}} \to b_{\tilde{a}} + \alpha\nabla g_{\tilde{a}}\ ,\ b_{\tilde{a}} \to g_{\tilde{a}} + \alpha\nabla b_{\tilde{a}}]{\varphi \to -\varphi + \alpha\nabla\varphi\ ,\ \bar{b}_{\tilde{a}\tilde{b}} \to \bar{b}_{\tilde{a}\tilde{b}} + \alpha\nabla\bar{b}_{\tilde{a}\tilde{b}}} S_{p-1}^{0wt}$: ΔS_0^1

Correction of T-Duality

$(\ \lambda_0 = -\frac{1}{8}\ )$

$$\begin{cases}
\Delta\varphi = \lambda_0[2\nabla^{\tilde{\mu}}\varphi\nabla_{\tilde{\mu}}\varphi + e^{\varphi}V^{\tilde{\mu}\tilde{\nu}}V_{\tilde{\mu}\tilde{\nu}} + e^{-\varphi}W^{\tilde{\mu}\tilde{\nu}}W_{\tilde{\mu}\tilde{\nu}}] \\
\Delta g_{\tilde{\mu}} = \lambda_0[2W_{\tilde{\mu}\tilde{\nu}}\nabla^{\tilde{\nu}}\varphi + e^{\varphi}\bar{H}_{\tilde{\mu}\tilde{\nu}\tilde{\alpha}}V^{\tilde{\nu}\tilde{\alpha}}] \\
\Delta b_{\tilde{\mu}} = \lambda_0[2V_{\tilde{\mu}\tilde{\nu}}\nabla^{\tilde{\nu}}\varphi - e^{-\varphi}\bar{H}_{\tilde{\mu}\tilde{\nu}\tilde{\alpha}}W^{\tilde{\nu}\tilde{\alpha}}] \\
\Delta\bar{b}_{\tilde{\mu}\tilde{\nu}} = \lambda_0[4V_{\tilde{\alpha}[\tilde{\mu}}W^{\tilde{\alpha}}{}_{\tilde{\nu}]} + 2g_{[\tilde{\nu}}W_{\tilde{\mu}]\tilde{\alpha}}\nabla^{\tilde{\alpha}}\varphi + 2b_{[\tilde{\nu}}V_{\tilde{\mu}]\tilde{\alpha}}\nabla^{\tilde{\alpha}}\varphi + e^{\varphi}g_{[\tilde{\nu}}\bar{H}_{\tilde{\mu}]\tilde{\alpha}\tilde{\beta}}V^{\tilde{\alpha}\tilde{\beta}} - e^{-\varphi}b_{[\tilde{\nu}}\bar{H}_{\tilde{\mu}]\tilde{\alpha}\tilde{\beta}}W^{\tilde{\alpha}\tilde{\beta}}]
\end{cases}$$

Field Strengths : $W_{\tilde{\mu}\tilde{\nu}} = \partial_{\tilde{\mu}}b_{\tilde{\nu}} - \partial_{\tilde{\nu}}b_{\tilde{\mu}}\ ,\ V_{\tilde{\mu}\tilde{\nu}} = \partial_{\tilde{\mu}}g_{\tilde{\nu}} - \partial_{\tilde{\nu}}g_{\tilde{\mu}}\ ,\ \bar{H}_{\tilde{\mu}\tilde{\nu}\tilde{\rho}} \equiv 3\partial_{[\tilde{\mu}}\bar{b}_{\tilde{\nu}\tilde{\rho}]} - \frac{3}{2}g_{[\tilde{\mu}}W_{\tilde{\nu}\tilde{\rho}]} - \frac{3}{2}b_{[\tilde{\mu}}V_{\tilde{\nu}\tilde{\rho}]}$

Correction DBI

- Expand DBI Action : With T-Duality Correction → $\Delta S_0^1 = -2\pi T_p \int d^p\sigma e^{-\bar{\phi}-\frac{\varphi}{4}} \sqrt{-\det(A^0_{\tilde{a}\tilde{b}})}[\frac{1}{4}\Delta\varphi + \frac{1}{2}Tr(A_0)^{-1}A^1]$

$$\begin{cases} A^0_{\tilde{a}\tilde{b}} = \bar{g}_{\tilde{a}\tilde{b}} + \bar{b}_{\tilde{a}\tilde{b}} + b_{[\tilde{a}}g_{\tilde{b}]} + e^{\varphi}g_{\tilde{a}}g_{\tilde{b}} \\ A^1_{\tilde{a}\tilde{b}} = \Delta\bar{b}_{\tilde{a}\tilde{b}} + b_{[\tilde{a}}\Delta b_{\tilde{b}]} + g_{[\tilde{a}}\Delta g_{\tilde{b}]} + 2e^{\varphi}g_{\{\tilde{a}}\Delta b_{\tilde{b}\}} - e^{-\varphi}g_{\tilde{a}}g_{\tilde{b}}\Delta\varphi \end{cases}$$

Reduction of Action(α′)

Reduction Action (WV) : S^{1w}_{p-1} → Buscher Rules ($\varphi \to -\varphi\,,\, b \leftrightarrow g$) → S^{1wT}_{p-1}

Reduction Action (Tra) : S^{1t}_{p-1}

$$\Delta S_1^0 = S^{1wT}_{p-1} - S^{1t}_{p-1}$$

T-Duality Constraint

- Total Derivative
  - $\tilde{J} = -\frac{\alpha' T_{p-1}}{2}\int d^p\sigma \sqrt{-\tilde{g}}\ \nabla_{\tilde{a}}\ (e^{-\bar{\phi}-\frac{\varphi}{4}} I^{\tilde{a}})$
  - all contraction($\bar{b}_{\tilde{a}\tilde{b}}, b_{\tilde{a}}, g_{\tilde{a}}, \bar{\phi}, \varphi$)

Constraint T-Duality: $\Delta\tilde{S}_0^1 + \Delta\tilde{S}_1^0 + \tilde{J} = 0$

- Result
- m=0 →
  - Parameters:
    - $a_2 = -2,\quad a_8 = -1 + \frac{a_1}{2},\quad b_1 = \frac{1}{24},\quad b_{11} = \frac{1}{6} - \frac{a_1}{6},\quad b_2 = -\frac{1}{4}$
    - $b_4 = \frac{1}{4},\quad b_{42} = -2 + 2a_1,\quad b_{45} = -1 + a_1,\quad b_{47} = 2 - a_1,\ b_5 = 1$
  - T D Vector : $\tilde{I}^{\tilde{a}} = -2e^{\varphi} g_{\tilde{b}} \partial^{\tilde{a}} g^{\tilde{b}} - \partial^{\tilde{a}}\varphi + (2 - a_1) e^{\varphi} g_{\tilde{b}} \partial^{\tilde{b}} g^{\tilde{a}} + a_1 e^{\varphi} g^{\tilde{a}} \partial^{\tilde{b}} g_{\tilde{b}}$
- a_1 Free →
  - Without boundary —(if $a_1 = 2$)→ Consistent S-Matrix [26,25]
  - with Boundary → fix $a_1 = 0$

Boundary Couplings

- Geometry : (Bulk + Boundary) : $M^{(26)}$, $\partial M^{(26)}$

- Boundary Action : $\partial S_p^1 = -\frac{\alpha' T_p}{2} \int_{\partial M^{(p)}} d^p \tau e^{-\phi} \sqrt{|\hat{g}|} \sum_{m=0}^{\infty} \partial L_m$

$$\hat{g}_{\hat{a}\hat{b}} = \frac{\partial \sigma^a}{\partial \tau^{\hat{a}}} \frac{\partial \sigma^b}{\partial \tau^{\hat{b}}} \tilde{G}_{ab}$$ Pull back of Pull Back metric

m=0 : $\partial L_0 = c_1 K_{ab} \tilde{G}^{ab}$

Extrinsic Curvature :

$$K_{\mu\nu} = \nabla_\mu n_\nu - n_\mu n^\rho \nabla_\rho n_\nu$$

Symmetric

$$n^\mu K_{\mu\nu} = 0$$

n_μ : Normal Vector Boundary $\partial M^{(26)}$

Boundary Couplings

Reduction

WV : $\partial L_0^w = c_1(\hat{K}_{\tilde{a}b}\bar{g}^{\hat{a}\tilde{b}} + \frac{1}{2}n_{\tilde{a}}\partial^{\tilde{a}}\varphi)$

Tra : $\partial L_0^t = c_1(\hat{K}_{\tilde{a}b}\bar{g}^{\hat{a}\tilde{b}} + e^{\varphi}n_{\tilde{a}}g_{\tilde{b}}\partial^{\tilde{a}}g^{\tilde{b}} - e^{\varphi}n_{\tilde{b}}g_{\tilde{a}}\partial^{\tilde{a}}g^{\tilde{b}})$

Total Derivative : $\tilde{J}(0) = \frac{\alpha' T_{p-1}}{2}\int_{\partial M^{(p)}} d^{p-1}\tau e^{-\phi}\sqrt{|g|}n_{\tilde{a}}[2e^{\varphi}g_{\tilde{b}}\partial^{\tilde{a}}g^{\tilde{b}} + \partial^{\tilde{a}}\varphi - (2-a_1)e^{\varphi}g_{\tilde{b}}\partial^{\tilde{b}}g^{\tilde{a}} - a_1 e^{\varphi}g^{\tilde{a}}\partial^{\tilde{b}}g_{\tilde{b}}]$

T-Duality Constraint on Boundary : $\partial S_p^{wT}(0) - \partial S_{p-1}^t(0) + \tilde{J}(0) = 0$ $\Longrightarrow$ $a_1 = 0$, $c_1 = 2$

Action(m=0)

$$S_p^1 + \partial S_p^1 = -\frac{\alpha' T_p}{2}\Big(\int d^{p+1}\sigma e^{-\phi}\sqrt{-\tilde{G}}[R^{ab}{}_{ab} + \frac{1}{6}H^{abc}H_{abc} + \frac{1}{4}H^{ab\mu}H_{ab\mu} - \frac{1}{4}H^{a\mu\nu}H_{a\mu\nu} + \frac{1}{24}H^{\mu\nu\alpha}H_{\mu\nu\alpha}$$

$$-2\Omega_{\mu ab}\Omega^{\mu ab} - \tilde{\nabla}_a\tilde{B}_{bc}\tilde{\nabla}^a\tilde{B}^{bc} + 2\nabla_a\phi\nabla^a\phi - 2\Omega_\mu{}^a{}_a\nabla^\mu\phi - \nabla_\mu\phi\nabla^\mu\phi] - \frac{\alpha' T_p}{2}\Big(\int d^p\tau e^{-\phi}\sqrt{-\tilde{g}}[2K_a^a]$$

T-Duality Constraint

m=1,2,3,4

Constraint : $\sum_{\tilde{m}=2}^{5} \Delta\tilde{S}_1^0(\tilde{m}) + \sum_{\tilde{m}=1}^{5} \Delta\tilde{S}_0^1(\tilde{m}) + \sum_{\tilde{m}=1}^{5} \tilde{J}(\tilde{m}) = 0$

Result:
- 65 Parameter
- 32 relation →

$a_7 = -1/2$, $b_{10} = -1/16$, $b_{12} = 1/2 - a_6/4$, $b_{13} = -2$, $b_{14} = 1/16$, $b_{15} = 1/4$,
$b_{27} = -2$, $b_3 = 1/96$, $b_{31} = 2$, $b_{32} = -1/2$, $b_{35} = b_{28}$, $b_{36} = -1 - b_{28}$, $b_{37} = b_{33}$,
$b_{38} = b_{34}$, $b_{39} = -8b_{25} - b_{33}$, $b_{40} = -8b_{26} - b_{34}$, $b_{44} = -1/2 + 2a_4$, $b_{46} = -1/4 + a_4$,
$b_{48} = -2$, $b_{49} = 1/2 - a_4$, $b_{50} = 0$, $b_{51} = -2$, $b_{55} = -2b_{24} + b_{41}$, $b_{56} = b_{23} + 16b_{25}$,
$b_{57} = -a_{14} + b_{23}$, $b_{58} = b_{24} + 16b_{26}$, $b_{59} = -b_{23} - 4b_{24}$, $b_6 = -1/4 + a_6/8$,
$b_{60} = -2a_{15} + b_{24}$, $b_{61} = -b_{23}$, $b_8 = -1/2 + b_7$, $b_9 = 2 - 4b_7$

DBI Factor

- Comparison Couplings

$$L_2(b_{22},a_4,b_{49},a_{12},b_{44},b_{46})=\frac{1}{4}B_{\tilde{a}\tilde{b}}B^{\tilde{a}\tilde{b}}L_0(b_{11},a_1,b_{47},a\ \ ,b_{42},a_{45})$$

$$L_3(b_{32},b_{34},b_{40},b_{58},b_{38},b_{60})=\frac{1}{4}B_{\tilde{a}\tilde{b}}B^{\tilde{a}\tilde{b}}L_1(b_{27},b_{28},b_{36},b_{50},b_{35},b_{51})$$

- Extra Factor : $B_{\tilde{a}\tilde{b}}B^{\tilde{a}\tilde{b}}$

$$L_2(b_{14},b_{10},b_3,b_{15},a_7,b_{22},a_4,b_{49},a_{12},b_{44},b_{46})=\sqrt{\det(1+\tilde{G}^{ab}\tilde{B}_{bc})}\ \ L_0$$

$$L_3(b_{32},b_{34},b_{40},b_{58},b_{38},b_{60})=\sqrt{\det(1+\tilde{G}^{ab}\tilde{B}_{bc})}\ \ L_1$$

All Contraction with Factor $B_{\tilde{a}\tilde{b}}B^{\tilde{a}\tilde{b}}$: $S_p^1=-\frac{\alpha' T_p}{2}\int d^{p+1}\sigma\sqrt{\det\tilde{G}_{ab}}\sum_{m=0}^{\infty}L_m$

Expand DBI factor $\sqrt{-\det(\tilde{G}_{ab}+\tilde{B}_{ab})}$: $S_p^1=-\frac{\alpha' T_p}{2}\int_{M^{(p+1)}}d^{p+1}\sigma e^{-\phi}\sqrt{-\det(\tilde{G}_{ab}+\tilde{B}_{ab})}\sum_{m=0}^{\infty}L_m$

Correction Propagators

- S-Matrix Consistent : ($a_1 = 2$) [26] ? → ($a_1 = 0$)

$$
\begin{cases}
L_0 = (\frac{1}{6} - \frac{a_1}{6}) H_{abc} H^{abc} + a1 \Omega_{\mu}{}^{b}{}_{b} \Omega^{\mu a}{}_{a} - 2\Omega_{\mu ab} \Omega^{\mu ab} + (-1 + \frac{a_1}{2}) \nabla_{\tilde{a}} (\tilde{B}_{bc} + \tilde{F}_{\tilde{b}\tilde{c}}) \nabla^{\tilde{a}} (\tilde{B}^{bc} + \tilde{F}^{bc}) \\
+ (2 - a_1) \nabla_a \phi \nabla^a \phi - (2 - a_1) \Omega_{\mu}{}^{a}{}_{a} \nabla^{\mu} \phi - (1 - a_1) \nabla_{\mu} \phi \nabla^{\mu} \phi
\end{cases}
$$

- Extra Contact term (Channel s)($s = -k^a k_a$)

Propagators (Without Correction)($a_1 = 2$)

- Gauge Fields → $(G^{XX})^{ij} = \frac{i\eta^{ij}}{T_p s}$
- Transvers Scalar Fields → $(G^{AA})^{ab} = \frac{i\eta^{ab}}{T_p s}$

Propagators (With Correction) ($a_1 = 0$)

- Gauge Fields → $(G^{XX})^{ij} = \frac{i\eta^{ij}}{T_p (1 - \alpha' s)} = \frac{i\eta^{ij}}{T_p} + \alpha' \frac{i\eta^{ij}}{T_p} + \ldots$
- Transvers Scalar Fields → $(G^{AA})^{ab} = \frac{i\eta^{ab}}{T_p (1 - \alpha' s)} = \frac{i\eta^{ab}}{T_p} + \alpha' \frac{i\eta^{ab}}{T_p} + \ldots$

Correction Propagators

- Contact term
  - 2 Dilaton
  - 2 B-field

$L_0(a_1 = 0)$ + Contact term = $L_0(a_1 = 2)$

Consistent S-Matrix Element

- $a_1 = 0$
- $a_1 = 2$

Conclusion

- Action D-Brane
  - m=0 (fix 10 Coupling + a_1 free) —— with boundary ——→ $a_1 = 0$
  - m > 1,2,3,4 (67 Coupling + 32 Relation)

- Special Background :(SFF)
  - Diagonal Metric
  - $\partial g_{\tilde{a}} \neq 0$
  - $g_{\tilde{a}} = 0$

Future works

Find Action For m > 4 ——→ Fix some Couplings m < 4

Find Covariant Form (Without Use Static Gauge) → (Background Arbitrary)

Check Propagators Receive α'^2 -Corrections → (Superstring theory)

Thanks