- What is the definition of the propagator $G(x,t,x_1,t_1)$ in a quantum theory. What is the physical meaning?

- Derive the propagator of a one-dimensional quantum harmonic oscillator of mass m and frequency ω_0 in coordinate basis:

$$G(x,t,x_1,0) = \frac{1}{i\hbar}\left(\frac{m\omega_0}{2\pi i\hbar\sin\omega_0 t}\right)^{\frac{1}{2}} \exp\left(-\frac{m\omega_0((x^2+x'^2)\cos\omega_0 t - 2xx')}{2i\hbar\sin\omega_0 t}\right). \quad (1)$$

- Transform the propagator into imaginary time $t \to -i\beta$ (Euclidean version of (1)) and obtain the equilibrium density matrix $\rho(x,x_1)$. Explain why β plays the role of inverse tempreture $\beta = 1/T$ and why is it periodic.

- Simplify the expression of $\rho(x,x_1)$ in the low $T \ll \hbar\omega_0$ and high $T \gg \hbar\omega_0$ temperature.

- Use the Euclidean version of (1) and derive the partition function of the theory in terms of it.

- Compare your result for the partition function with

$$Z = \sum_{n=0}^{\infty} e^{-\beta E_n} \quad (2)$$

  where $E_n = \hbar\omega\left(n+\frac{1}{2}\right)$ are the energy levels of the harmonic oscillator.

- Derive the ground state energy and the wave function of the harmonic oscillator by taking an appropriate limit in the the Euclidean version of (1).