- Starting with path integral for the real free-field theory

$$Z_0[J] = \int \mathcal{D}\varphi e^{i\int d^4x(\mathcal{L}_0 + J\varphi)} \tag{1}$$

where

$$\mathcal{L}_0 = -\frac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \frac{1}{2}m^2\varphi^2 \tag{2}$$

Show that

$$Z_0(J) = \exp\left(\frac{i}{2}\int \mathrm{d}^4x\, \mathrm{d}^4y J(x)\Delta(x-y)J(y)\right), \tag{3}$$

where $\Delta(x-y)$ is the Feynman propagator.

- Consider the interacting quantum field theory with $\mathcal{L}_{int} = -\frac{\lambda}{4!}\varphi^4$. Write the normalization conditions such that the LSZ formula is valid. Write the full Lagrangian $\mathcal{L}$ in terms of the parameters $Z_\varphi, Z_m, Z_\lambda$. What is the counterterm Lagrangian?

- What is the mass dimension of the field φ and the coupling constant λ in general dimensions. Start with the path-integral for this theory and determine F such that

$$Z(J) = \int \mathcal{D}\varphi e^{i\int \mathrm{d}^4x(\mathcal{L}+J\varphi)} \propto F\left(\frac{1}{i}\frac{\delta}{\delta J}\right) Z_0(J) \tag{4}$$

- Use the Feynman diagrams, ignoring the counter terms, draw all the connected diagrams with at most 4 sources and 2 vertices.

- Draw all 1PI diagrams.

- Compute the order-λ correction to the propagator in this theory. (Do not do the integrals)

- Compute the $\mathcal{O}(\lambda^2)$ correction to the exact vertex.