Yukawa Theory

- Consider the Yukawa Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi - \frac{1}{2}M^2\varphi^2 + g\varphi\bar{\psi}\psi \tag{1}$$

- Use the Feynman rules to write down (at tree level) $i\mathcal{T}$ for the process $e^-e^- \to e^-e^-$ and $\varphi\varphi \to e^-e^+$.

- **Home** Compute the spin-averaged cross section $\langle|\mathcal{T}|^2\rangle$ for $e^-e^- \to e^-e^-$ and $\varphi\varphi \to e^-e^+$. Use the crossing symmetry to find $\langle|\mathcal{T}|^2\rangle$ for other related processes.

- Add all relevant or marginal possible terms that respect the symmetries of the original Lagrangian (1). Argue why renormalizability of the theory requires the inclusion of these terms.

- Write the improved Lagrangian in terms of the renormalized parameters and the counterterms.

- Write the bare Lagrangian and find the relation between the bare and renormalized couplings in terms of the leading renormalized parameters.

- Draw the diagrams for the one-loop and counterterm corrections to the scalar and fermion propagator and vertex. (Do not do the integrals)

- **Home** Compute the one-loop contribution to the beta functions and to the anomalous dimensions.

Photons

- Consider the Lagrangian for a free massless vector field modified with a gauge fixing term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\xi\left(\partial_\lambda A^\lambda\right)^2 \tag{2}$$

- Derive the following equal-time commutation relations

$$[A^\mu(t,\vec{x}), A^\nu(t,\vec{y})]\,, \quad [A^\mu(t,\vec{x}), \dot{A}^\nu(t,\vec{y})]\,, \quad [\dot{A}^\mu(t,\vec{x}), \dot{A}^\nu(t,\vec{y})]\,, \tag{3}$$

- **Home** Derive the propagator of the theory.

Scalar QED

- Write down the Lagrangian density of a U(1) gauge theory coupled to two complex massive scalar fields ϕ and χ with the same coupling constant e.
- **Home** Perform the perturbation theory at $\mathcal{O}(e)$ to find the time ordered 4-point functions that allows interaction with the gauge field A_μ. (Feynman diagrams)

Ward identity

- In the spinor electrodynamics write down the tree level 2-point and 3-point momentum space correlation functions $S(p_1, p_2)$ and $V_\mu(p_1, p_2, k)$ where p's are the fermion momentum and k the photon momentum.
- Show that $\partial S(p)/\partial p_\mu$ are related to $V_\mu(p, -p, 0)$ (at tree-level) and argue why this is a consequence of gauge symmetry.
- **Home** Verify the same relation between the 2-point and the 3-point vertex function at one-loop level.

Non-Abelian gauge theory

- Write down the Lagrangian density for a non-Abelian gauge theory with gauge group G coupled to a complex scalar field in representation R.
- **Home** Compute the contribution of this scalar field to the β-function, and show that the full beta function for this theory is

$$\beta(g) = -\frac{g^3}{16\pi^2}\left(\frac{11}{3}T(A) - \frac{1}{3}T(R)\right) . \tag{4}$$