- Consider the pseudoscalar Yukawa lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + i\bar{\psi}\partial\!\!\!/\psi - m\bar{\psi}\psi - \frac{1}{2}M^2\varphi^2 + ig\varphi\bar{\psi}\gamma_5\psi \tag{1}$$

- Find the internal symmetry of the Lagrangian and the corresponding Noether current and the conserved charge Q. Compute the Poisson bracket of the charge Q with the fields.
- Write the transformation of the fields such that the Lagrangian is invariant under parity.
- Add all relevant or marginal possible terms that respect the symmetries of the original Lagrangian (1). Argue why the theory can not be renormalized without the inclusion of such terms.
- Write the improved Lagrangian in terms of the renormalized parameters and the counterterms.
- Derive the Feynman rules in terms of the renormalized parameters.
- Draw and compute the one-loop and counterterm corrections to the scalar propagator. (Do not do the integrals)
- Draw and compute the one-loop and counterterm corrections to the fermion propagator. (Do not do the integrals)
- Draw and compute the $\mathcal{O}(g^3)$ correction to the scalar-fermion-fermion vertex.
- Draw and compute the one-loop correction to the scalar vertex. (Do not do the integrals)
- Write the bare Lagrangian and find the relation between the bare and renormalized

couplings in terms of the leading renormalized parameters which are as follows,

$$Z_\psi = 1 - \frac{g^2}{16\pi^2}\left(\frac{1}{\varepsilon} + \text{finite}\right) , \tag{2}$$

$$Z_\varphi = 1 - \frac{g^2}{4\pi^2}\left(\frac{1}{\varepsilon} + \text{finite}\right) , \tag{3}$$

$$Z_g = 1 + \frac{g^2}{8\pi^2}\left(\frac{1}{\varepsilon} + \text{finite}\right) , \tag{4}$$

$$Z_\lambda = 1 + \left(\frac{3\lambda}{16\pi^2} - \frac{3g^2}{\pi^2\lambda}\right)\left(\frac{1}{\varepsilon} + \text{finite}\right) . \tag{5}$$

Compute the beta function $\beta(g)$ and $\beta(\lambda)$ to first order in the couplings, assuming λ and g^2 are of the same order.