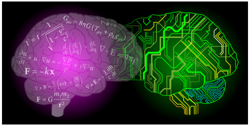
ML and Physics

Lec II: ML for Physics



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What is Physics?

- A Physical law:

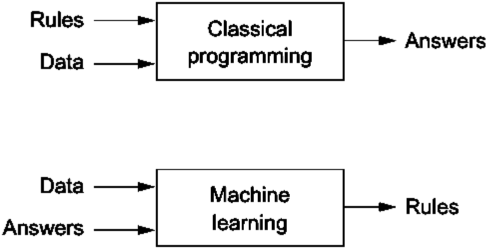
$$\mathcal{S}_{\mathrm{i}} \xrightarrow[\Delta \mathrm{t}]{\mathscr{L}} \mathcal{S}_{\mathrm{f}} \ ,$$

- A restriction on the space of (mathematically) possible S
- How we make theories?
- The whole Process:
  1. Information from nature
  2. (Complicated) process of analysis by the physicist (intelligence)
  3. Making Models
  4. Prediction of new Phenomena
- In Brief: Data $\Rightarrow$ Training (intelligence) $\Rightarrow$ Predictions

The Physics Challenges

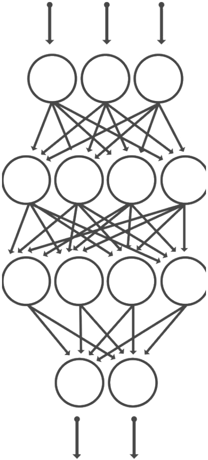
1. What are good representations of States $\mathcal{S}$ and Laws $\mathcal{L}$?
   - The representations $(\mathcal{S}, \mathcal{L})$ are usually (in practice and maybe in principle) effective
2. Assuming Universality of a given description, Applying it to different systems
   - Description of larger systems in term of smaller systems (Different Level of effectiveness)
   - Solving models
3. Relation of description with Previous Knowledge and Models

Two Paradigms for Programming

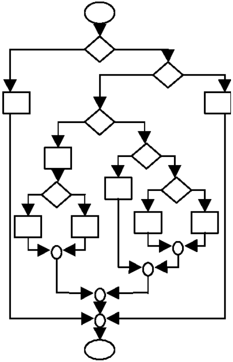


- Hard problems: a lot of computational steps (time) or/and memory
- Finite amount of resource

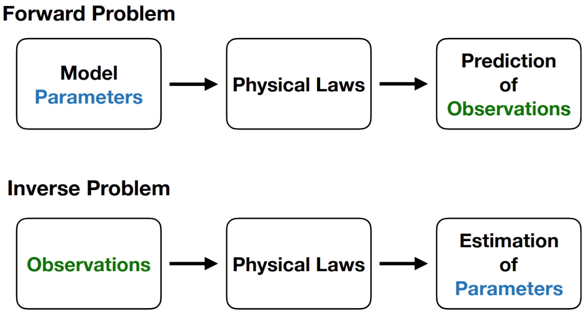
Two Computations



```
IF B > D THEN
  IF A > B THEN
    PRINT C * C
  ELSE
    IF C > A THEN
      IF D > C THEN
        IF B > C THEN
          PRINT B * C
        ELSE
          PRINT C * D
        END IF
      ELSE
        PRINT A * B
      END IF
    ELSE
      PRINT A * C
      IF A > D THEN
        PRINT A * D
      ELSE
        PRINT B * D
      END IF
    END IF
  END IF
ELSE
  PRINT A * A
END IF
```



Inverse problems and ML



- e.g. Discovery of Neptune from the perturbed trajectory of Uranus.
- Earth density from acoustic wave
- Tomographic reconstruction, i.e CT scan

An Inverse Problem

- Given the input x find the value of f(x) by:

$$\mathrm{f(x)} = \sum_{\mathrm{k}=0}^{\mathrm{n}} \mathrm{J_k x^k}$$

- The Inverse:
  1. given an output $\mathrm{y_1}$ find the input $\mathrm{x_1}$ such that $\mathrm{y_1 = f(x_1)}$
  2. given some data $\{(\mathrm{x_1, y_1}), (\mathrm{x_2, y_2}), \dots\}$ find the function $\mathrm{y = f(x)}$, i.e. $\mathrm{J_k}$

- Example:

$$\mathrm{y_i} = \sum_{\mathrm{k}=1}^{\mathrm{m}} \mathrm{J_{ik} x_k}$$

- Very special cases: $(\mathrm{n} = \mathrm{m}, \det \mathrm{J} \neq 0)$

$$\mathrm{x_k} = \sum_{\mathrm{i}=1}^{\mathrm{m}} \left[\mathrm{J}^{-1}\right]_{\mathrm{ki}} \mathrm{y_i}$$

- In many cases, the solution to the inverse problem is ill-posed.
- In general it is possible to define:

$$\mathrm{L} \equiv \sum_{\mathrm{i}=1}^{\mathrm{n}} \left(\mathrm{y_i} - \sum_{\mathrm{k}=1}^{\mathrm{m}} \mathrm{J_{ik} x_k}\right)^2 = 0$$

- seek an $\mathrm{x_k}$ that minimizes L

Inverse Problem And ML

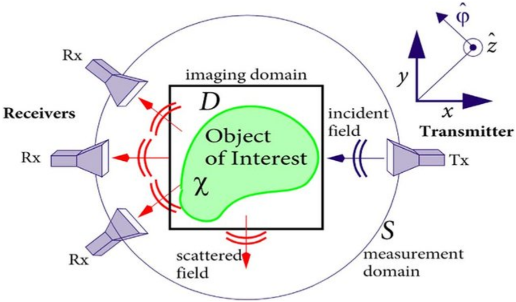
- $\Rightarrow$ Optimization problem
- $\Rightarrow$ Representing the map with an NN
- Estimation of parameters of the model based on observation
- Printing Vs Digit Recognition

Inverse Problems in Physics

- Knowing objects that cannot be measured directly
- Infering the cause from the results
- Determining physical laws and governing equations
- Determining physical constants
- All physics innovations are inverse problems

Scattering And Imaging

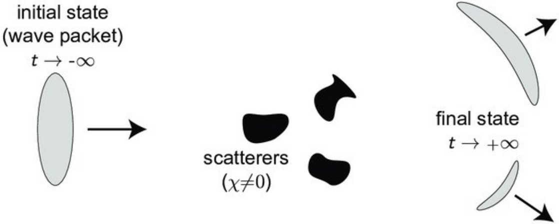
- Unknown Object



- The image is reconstructed from information of scattered waves

Inverse Scattering Problem

- Unknown Potential, Law,



- The potential is determined from information such as scattering amplitudes

Simulations and ML

- Use of computer simulations to generate samples of labeled training data $\{x_i, y_i\}$
- Learn fast neural network to generate the results of simulations.
- Data generation
- Learn the network once

Machine learning phases of matter[1, 2]

- Large state space $\rightarrow$ complexity
- Classification of phases
- Identify phases from state
- $\Rightarrow$ neural-network classification
- Study of phase transitions and order parameters
- Classify The Ising Phases

[1] nature.com/articles/nphys4035
[2] nature.com/articles/nphys4037

String Theory And ML

- String Theory: First Prediction: 10d spacetime
- Don't Worry $\Rightarrow \mathrm{M}^{10} = \mathrm{M}^4 \times \mathrm{X}^6$
- A lot of possibilities! ☹
- Topology and geometry of compact dimensions (in addition of Branes and Fluxes)
- The problem is to select a manifold necessary for compaction of string theory so that it becomes the standard model of elementary particles
- $\Rightarrow$ An inverse problem

2-D Manifolds

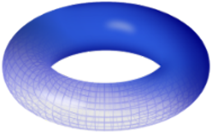
 . . .

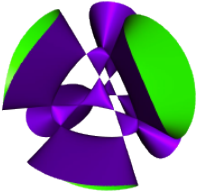
$g(\Sigma) = 0$	$g(\Sigma) = 1$	$g(\Sigma) > 1$
$\chi(\Sigma) = 2$	$\chi(\Sigma) = 0$	$\chi(\Sigma) < 0$
Spherical	Ricci-Flat	Hyperbolic
$+$ curvature	0 curvature	$-$ curvature

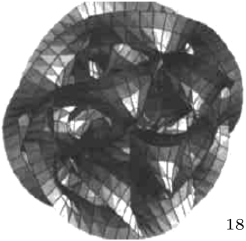
String Phenomenology to CY

- Classical vacuum of string theory on X^{10} preserves $\mathcal{N} = 1$SUSY in M^4
- Killing Spinor $\Rightarrow$ Covariant constant spinor $\Rightarrow$ Ricci-Flat internal X^6 space
- The gauge group/particle content of low energy theory depends on the topology of compact space
- Direct Problem: Find the resulting 4D theory from a given topology?

Calabi–Yau manifolds

$d = 1$ Torus $T^2 = S^1 \times S^1$ 

$d = 2$ K3  ; 4-torus: $T^4 = \left(S^1\right)^4$

$d = 3$ CY3: Unclassified, billions known 

Complete Intersection Calabi-Yau

- CY as Intersections of hypersurfaces in $\mathbb{P}^n$
- Configuration matrices:

$$K = \left[\begin{array}{c|cccc} n_1 & q_1^1 & q_1^2 & \dots & q_1^K \\ n_2 & q_2^1 & q_2^2 & \dots & q_2^K \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ n_m & q_m^1 & q_m^2 & \dots & q_m^K \end{array}\right]_{m\times K}$$

- Classifying the CICY matrices, checking redundancies and equivalences
- Obtained $\sim 10^{10}$ configuration

Train The CYs

- Training data: $\{\mathrm{k_i}, \mathrm{h}\,(\mathrm{k_i})\}$ from very lengthy calculations
- Being Similar to SM as a label:

SM

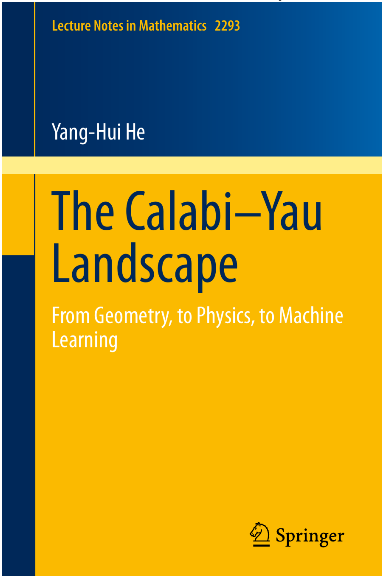
$$K = \begin{pmatrix} -1 & -1 & -1 & 1 & 2 \\ 0 & -2 & 0 & 1 & 1 \\ -1 & 1 & -1 & 0 & 1 \\ 1 & 0 & 1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -1 \\ 1 & 0 & 1 & -2 & 0 \end{pmatrix} \to 1$$

non-SM

$$K = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 2 & 2 & -1 & -2 \\ 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 1 & -1 \\ 1 & -1 & 0 & -1 & 1 \end{pmatrix} \to 0$$

- Design the network $\Rightarrow$ Probe the landscape, More than 90% accuracy
- Refer to HRD problem

The CY Landscape: from Geometry to Physics, to ML

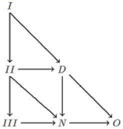


Machine-Learning the Classification of Spacetimes [1]

- Petrov-Penrose Classification of Spacetimes

$$\Psi_0 \equiv C_{abcd} k^a m^b k^c m^d, \quad \Psi_1 \equiv C_{abcd} k^a l^b k^c m^d$$
$$\Psi_2 \equiv C_{abcd} k^a m^b \bar{m}^c l^d \quad \Psi_3 \equiv C_{abcd} k^a l^b \bar{m}^c l^d$$
$$\Psi_4 \equiv C_{abcd} \bar{m}^a l^b \bar{m}^c l^d$$

$$\Psi_0 - 4z\Psi_1 + 6z^2\Psi_2 - 4z^3\Psi_3 + z^4\Psi_4 = 0$$



Petrov type	Multiplicities
I	(1,1,1,1)
II	(2,1,1)
D	(2,2)
III	(3,1)
N	(4)
O	-

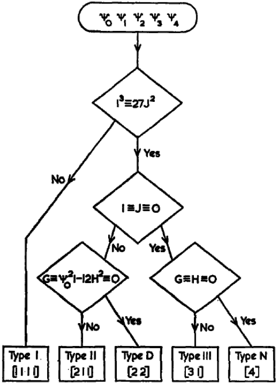


FIG. 1. Flow diagram for determining the Petrov type from the Ψ's.

- NNs: $\{\Psi_0, \Psi_1, \Psi_2, \Psi_3, \Psi_4\} \rightarrow$ Types

[1] arXiv:2201.01644 , Y. He, J. Manuel P. Ipiña

Particle Physics

- Experiments such as CMS and ATLAS at the LHC generate petabytes of data per year.
- Features x_i in detector labeled y_i by particles or interactions $\Rightarrow$ classification
- Signal and Background discrimination
- Jet Classification (heavy and light quarks, gluons, and W, Z, and H bosons)
- Fast Simulation: SM and BSM leads to which results
- Search for anomalies: classify SM and BSM events
- Neutrino physics, phase transitions of quantum chromodynamics, ...
- Many other applications[1], [2]

[1] arXiv:1807.02876
[2] arXiv:1806.11484

Cosmology And Astrophysics

- Gravitational lens finding
- Simulation
- Inverse problem: Interferometer Gravitational-Wave Observatory (LIGO) time series to the underlying waveform from a gravitational wave
- Data recorded on detectors $\rightarrow$ gravitational wave
- Reconstruct the image of a black hole from data from array of telescopes (EHT)
- Classification of galaxies
- ...[1]

[1] arXiv:2203.08056

Other Applications

- Other examples of inverse problems in physics
- Inverse of boundary value problem
- Landscape of Theories e.g. Conformal Bootstrap, Space of Integrable Theories
- Quantum State Tomography[1]: Reconstruct the density matrix of an unknown quantum state, through experimentally available measurements
- AdS/CFT correspondence: determining gravity theory, the inside (bulk), from the quantum field theory living on the boundary

[1] arxiv:1703.05334

Wave Function of a Quantum Many-Body System

- The Quantum state

$$|\psi\rangle = \sum_{\mathrm{s}_1,\cdots,\mathrm{s}_\mathrm{N}} \psi\left(\mathrm{s}_1,\cdots,\mathrm{s}_\mathrm{N}\right)|\mathrm{s}_1\rangle\cdots|\mathrm{s}_\mathrm{N}\rangle$$

- The nonlinear function ψ transforms the input $(\mathrm{s}_1, , \mathrm{s}_\mathrm{N}) = (0, 0, 1, 0, 1,)$ into output $\psi(0, 0, 1, 0, 1,)$.
- Minimize the energy function

$$\mathrm{E} = \frac{\langle\psi|\mathrm{H}|\psi\rangle}{\langle\psi \mid \psi\rangle}$$

- The ground state

The NN Quantum States

- Neural networks as a method of constructing quantum states

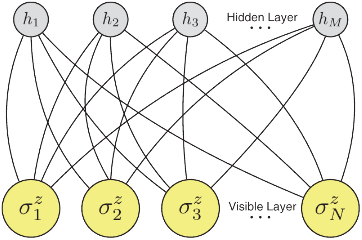
  Represent the nonlinear state function as $|\Psi\rangle$ by

$$\Psi(\mathrm{q}) \equiv \mathrm{g}^{(\mathrm{L})}\left(\theta^{(\mathrm{L})} \ldots \mathrm{g}^{(2)}\left(\theta^{(2)} \mathrm{g}^{(1)}\left(\theta^{(1)} \mathrm{q}\right)\right)\right)$$

- An Energy Function
- $\Rightarrow$ Optimization $\Rightarrow \theta^{\star}$
- $\Rightarrow \Psi_{\theta^{\star}}(\mathrm{q})$

RBM and The Wave Function[1]

- Restricted Boltzmann Machines:



- The Neural-Network Quantum States:

$$\Psi_{\mathrm{M}}(\mathcal{S};\mathcal{W}) = \sum_{\{\mathrm{h_i}\}} \mathrm{e}^{\sum_j \mathrm{a_j}\sigma_j^z + \sum_i \mathrm{b_i h_i} + \sum_{ij} \mathrm{W_{ij} h_i}\sigma_j^z}$$

- Minimize the Energy Function

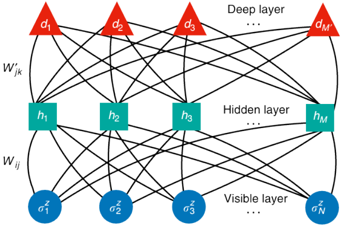
$$\mathrm{E}(\mathcal{W}) = \langle \Psi_{\mathrm{M}} | \mathcal{H} | \Psi_{\mathrm{M}} \rangle \,/\, \langle \Psi_{\mathrm{M}} \mid \Psi_{\mathrm{M}} \rangle$$

- Exponential to polynomial complexity

[1] Science 355, 602 (2017) 8.

DBM and The Wave Function[1]

- Deep Boltzmann Machines:



$$\Psi_{\mathcal{W}}\left(\sigma^{z}\right)=\sum_{\{h,d\}}\exp\left[\sum_{i}a_i\sigma_i^z+\sum_{ij}\sigma_i^z W_{ij}h_j+\sum_{j}b_j h_j+\sum_{jk}h_j d_k W'_{jk}+\sum_{k}b'_k d_k\right]$$

[1] Nat Commun 9, 5322 (2018).

Some Refrences

1. A high-bias, low-variance introduction to Machine Learning for physicists: arXiv:1803.08823
   - Introduction to the core concepts and tools of machine learning
   - Python Jupyter notebooks
2. Deep Learning and Physics; Akinori Tanaka, Akio Tomiya, and Koij Hashimoto, springer.com/book/10.1007/978-981-33-6108-9

Summery

- Technique and methods of ML are well adapted to physics
- Using ML in physics is rapidly expanding
- The range of problems vary from classical physics, to quantum theory, particle physics and string theory