Top quark; a source to search for new physics

S. Mohammad Moosavi Nejad

Yazd University

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Top Quark (History and Motivations)

- Light quarks (u, d, s) led us to the Parton Model of the SM
- From the Charm quark, we learned that the SM is a consistent theory. Moreover, its discovery confirmed QCD as the quantum theory of the strong interactions $\rightarrow J/\psi(^3S_1), \eta_c(^1S_0)$
- From the Bottom quark, we learned that a complete third family exists. Also, the weak CP violation is a part of the SM
- Top quark was discovered by the CDF and D0 experiments at the Tevatron in 1995
- Its remarkably large mass is still the largest of any known elementary particle: $m_t = 172.69 \pm 0.30$ GeV (from LHC and Tevatron Runs)
- Due to its high mass, it plays a crucial role in testing the electroweak symmetry breaking mechanism and in searching for new physics beyond the SM
- LHC is a superlative top factory, producing about 90 million top quark pairs per year of running at design c.m. energy $\sqrt{s} = 14$ TeV and design luminosity $10^{34} cm^{-2} s^{-1}$ in each of the four experiments

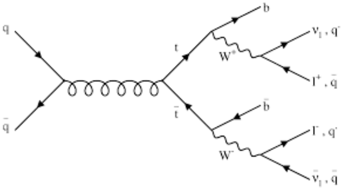
Top couplings in the Standard Model

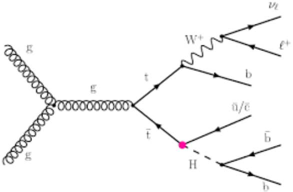
Various elementary interactions of top quark field $t(x^\mu)$ in the SM Lagrangian, read:

- The charged weak interaction of the top quark with other quarks is left-handed and flavor-changing:
$$\frac{g_w}{2\sqrt{2}} V_{tf}\,\bar{t}(x)\gamma^\mu(1-\gamma_5)f(x)W_\mu(x), \quad f = d, s, b$$
- Its neutral weak interaction is flavor-conserving and parity violating
$$\frac{g_w}{4\cos\theta_w}\bar{t}(x)\gamma^\mu[(1-\frac{8}{3}\sin^2\theta_w)-\gamma_5]t(x)Z_\mu(x)$$
- Its interaction with gluons is a vector-like coupling, involving an SU(3) generator (T^a) in the fundamental representation
$$g_s\bar{t}_i(x)\gamma^\mu T^a_{ij}t_j(x)G^a_\mu(x) \quad i,j = 1,2,3 \quad a = 1,2,\cdots 8$$
- Top interaction with photons is also simply vector-like as
$$\frac{2}{3}e[\bar{t}(x)\gamma^\mu t(x)A_\mu(x)]$$
- Its interaction with the Higgs field $h(x)$ is of the Yukawa type
$$y_t h(x)\bar{t}(x)t(x), \quad y_t = \sqrt{2}m_t/v, \quad v = < h >_0$$
- Effective interactions such as for flavour-changing neutral currents, occur due to loop corrections which are so small

Top quark features

- Top width is large ($\Gamma_t \approx 2\text{GeV} \rightarrow \tau_t = \hbar/\Gamma_t \approx 5 \times 10^{-25}$ s) $\rightarrow$ no hadronization takes place
- Rapid decay of the top quark enables transmission of its spin information into final states
- The width-to-mass ratio Γ_t/m_t of the top quark is small enough that, the notion of top quark as a stable particle makes sense
- Accurate measurements of CDF Collaboration of ratio $R = Br(t \rightarrow Wb)/Br(t \rightarrow Wq)$ $[q = d, s$ or $b]$:
  $V_{tb} \simeq 1 \Longleftrightarrow Br(t \rightarrow b + W) \simeq 1.$
- At LHC, the following production and decay modes are more likely:





Top quark decay in the SM: $t \to bW^+ \to bl^+\nu_l$

Off-shell W-boson

$$d\Gamma_0 = \frac{1}{2m_t}\overline{|M_0|^2}\frac{1}{(2\pi)^5}\frac{d^3\vec{p}_b}{2E_b}\frac{d^3\vec{p}_\nu}{2E_\nu}\frac{d^3\vec{p}_l}{2E_l}\delta^4(p_t - p_b - p_\nu - p_l)$$

where

$$|M_0^{SM}(t \to bl^+\nu_l)|^2 = \frac{g_W^4 m_t^3 |V_{tb}|^2}{(p_W^2 - m_W^2)^2 + m_W^2\Gamma_W^2} E_l \left\{1 + (\frac{m_l}{m_t})^2 - (\frac{m_b}{m_t})^2 - \frac{2E_l}{m_t}\right\}$$

$$\Gamma_0^{SM}(t \to b\tau^+\nu_\tau) = 0.1543 \quad \leftarrow m_l = 0$$

On-shell W-boson: Narrow width approximation

$\Gamma_W \ll M_W \to$ the Breit-Wigner Resonance is replaced by a delta-function

$$\frac{1}{(p_W^2 - m_W^2)^2 + (m_W\Gamma_W)^2} \approx \frac{\pi}{m_W\Gamma_W}\delta(p_W^2 - m_W^2)$$

$$\Gamma(t \to bl^+\nu_l) = \Gamma(t \to bW^+)\frac{\Gamma(W^+ \to l^+\nu_l)}{\Gamma_W} = \Gamma(t \to bW^+)B(W^+ \to l^+\nu_l)$$

$$\Gamma_0^{SM}(t \to b\tau^+\nu_\tau) = 1.463 \times Br(W^+ \to \tau^+\nu_\tau) = 0.1645 \Longrightarrow \frac{\Delta\Gamma_0}{\Gamma_0} = 6.6\%$$

$$\Gamma_0^{SM}(t \to b\mu^+\nu_\mu) = 0.1543$$

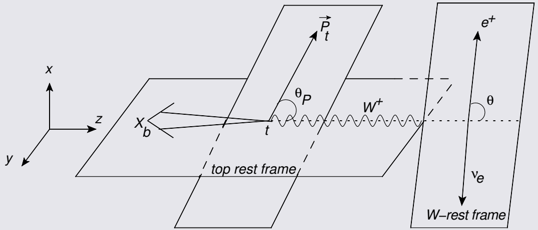
$$\Gamma_0^{SM}(t \to be^+\nu_e) = 0.1569$$

PRD 99, 095012 (2019)

Narrow width approximation factorizes the production and decay rates

Proposed channel to search: $t(\uparrow) \to bW^+ \to B + l^+\nu_l + X$

Definition of Top and W-rest frames



Triply differential partial decay width: Factorization theorem

$$\frac{d^3\Gamma}{dx_B\, d\cos\theta\, d\cos\theta_P} = \sum_{a=b,g} \int_{x_B}^{1} \frac{dx_a}{x_a} \frac{d^3\hat{\Gamma}_a}{dx_a\, d\cos\theta\, d\cos\theta_P}(\mu_R, \mu_F) D_a^B\left(\frac{x_B}{x_a}, \mu_F\right)$$

where

$$\frac{d^3\hat{\Gamma}_a(t\to bl^+\nu_l)}{dx_a\, d\cos\theta\, d\cos\theta_P} = \frac{1}{2}\left(\frac{d^2\hat{\Gamma}_a^{\text{unpol}}}{dx_a\, d\cos\theta} + P\frac{d^2\hat{\Gamma}_a^{\text{pol}}}{dx_a\, d\cos\theta}\cos\theta_P\right) \quad , \quad x_B = E_B/E_b^{max}$$

Wilson coefficients at NLO:

$$\frac{d^2\hat{\Gamma}_a}{dx_a\, d\cos\theta} = \frac{3}{8}(1+\cos\theta)^2\frac{d\hat{\Gamma}_a^+}{dx_a} + \frac{3}{8}(1-\cos\theta)^2\frac{d\hat{\Gamma}_a^-}{dx_a} + \frac{3}{4}\sin^2\theta\frac{d\hat{\Gamma}_a^0}{dx_a}$$

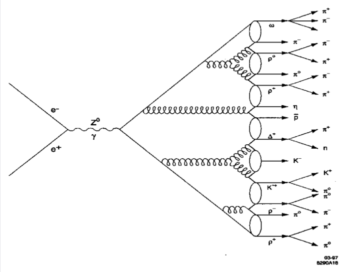
Fragmentation Functions in phenomenological approaches

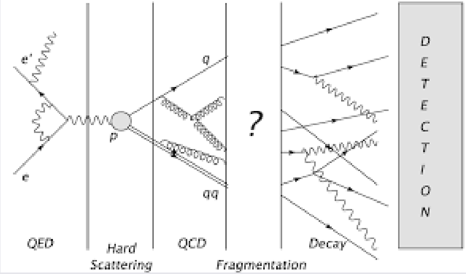
Non-perturbative fragmentation functions:

- Simple Power model: $D_i^H(x; \mu_0, \alpha, \beta) = N x^\alpha (1-x)^\beta$
- Peterson model: $D_i^H(x, \mu_0, \epsilon) = N \frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}$ $\qquad x = E_H/E_i$

Note: Free parameters are determined by fitting as

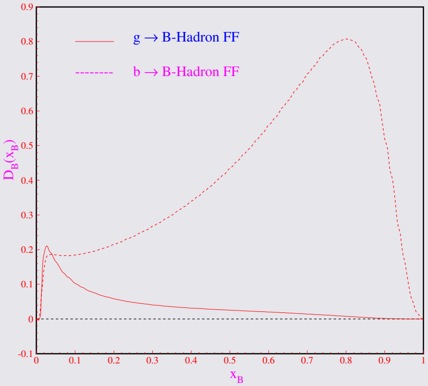
$$d\sigma^{Exp}(e^- e^+ \to q\bar{q} \to B + jet) = d\hat{\sigma}(e^- e^+ \to q\bar{q}) \otimes D_q^B$$





DGLAP evolution equations to get FFs at desired scale

$\frac{d}{d\log\mu_F^2} D_i(x_B, \mu_F, m_b) = \sum_j \int_{x_B}^{1} \frac{dz}{z} P_{ij}\left(\frac{x_B}{z}, \alpha_s(\mu_F)\right) D_j(z, \mu_F, m_b)$
where $P_{ij}(x_B)$ are well-known Altarelli-Parisi splitting functions.



$(g, b) \to B$ FFs at $\mu = m_t$ in the Power model fitted to the OPAL, ALEPH, SLD data at initial scale $\mu_0 = m_b$ (PRD 99, 114001 (2019)).

Custom approaches to compute Wilson functions in pQCD

Wilson coefficients:

$$\frac{d^2\hat{\Gamma}_a(t\to bW^+)}{dx_a\, d\cos\theta} = \frac{3}{8}(1+\cos\theta)^2\frac{d\hat{\Gamma}_a^+}{dx_a} + \frac{3}{8}(1-\cos\theta)^2\frac{d\hat{\Gamma}_a^-}{dx_a} + \frac{3}{4}\sin^2\theta\frac{d\hat{\Gamma}_a^0}{dx_a}$$

There are generally two approaches to compute the Wilson coefficients:

ZM-VFN scheme or Zero-Mass variable-flavor-number scheme:

1. The assumption $m_b = 0$ is adopted from the beginning
2. Partonic decay rate is free of large logarithms $\log(m_t^2/m_b^2)$
3. Singularities due to soft/hard emitted gluons and collinear ones

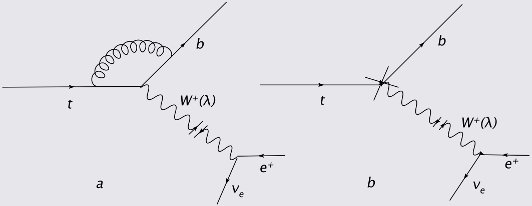
GM-VFN scheme or General Mass:

1. The b-quark mass is preserved during computations
2. Singularities are due to the soft and hard gluon emissions
3. Big problem: $\lim_{m_b\to 0} d\tilde{\Gamma}(m_b) \neq d\hat{\Gamma}(m_b = 0)$
4. Subtraction terms: $d\Gamma^{sub} = \lim_{m_b\to 0} d\tilde{\Gamma}(m_b) - d\hat{\Gamma}(m_b = 0)$
5. $d\hat{\Gamma}^{GM-VFN} = d\tilde{\Gamma}(m_b \neq 0) - d\Gamma^{sub}$

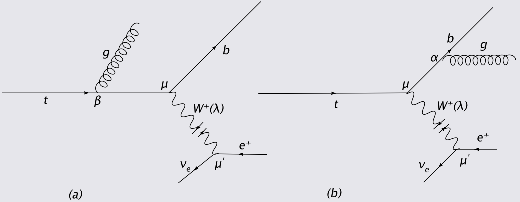
Wilson coefficients: Feynman Diagrams at NLO

$$\frac{d^3\Gamma(t\to BX)}{dx_B\, d\cos\theta\, d\cos\theta_P} = \sum_{a=b,g}\int_{x_B}^{1}\frac{dx_a}{x_a}\frac{d^3\hat{\Gamma}_a(t\to bW^+)}{dx_a\, d\cos\theta\, d\cos\theta_P}(\mu_R,\mu_F)D_a\left(\frac{x_B}{x_a},\mu_F\right)$$

Virtual Corrections:



Real Corrections:



Technical details

- To regulate IR and UV divergences dimensional regularization is used

$$\int \frac{d^4 p_g}{(2\pi)^4} \to \mu^{4-D} \int \frac{d^D p_g}{(2\pi)^D}$$

In this scheme, all divergences regularized in $D = 4 - 2\epsilon$ dimensions

- the renormalized amplitude of the virtual corrections is

$$M_{loop} = \frac{-e}{2\sqrt{2}\sin\theta_W} \epsilon^{\star}_{\mu}(p_W)\bar{u}(p_b, s_b)\{\Lambda_\mu + \delta\Lambda_\mu\}u(p_t, s_t)$$

- For the counter term of the vertex, one has:

$$\Lambda_{ct} = (\frac{\delta Z_b}{2} + \frac{\delta Z_t}{2} - \frac{\delta m_t}{m_t} - \frac{\delta m_b}{m_b})\gamma_\mu(1-\gamma_5)$$

  - The wave function and the mass renormalization constants reads

$$\delta Z_t = -\frac{\alpha_s(\mu_R)}{4\pi} C_F(\frac{1}{\epsilon_{UV}} + \frac{2}{\epsilon_{IR}} - 3\gamma_E + 3\ln\frac{4\pi\mu_F^2}{m_t^2} + 4),$$

$$\delta Z_b = -\frac{\alpha_s(\mu_R)}{4\pi} C_F(\frac{1}{\epsilon_{UV}} - \frac{1}{\epsilon_{IR}})$$

$$\frac{\delta m_q}{m_q} = \frac{\alpha_s(\mu_R)}{4\pi} C_F\left[\frac{3}{\epsilon_{UV}} - 3\gamma_E + 3\ln\frac{4\pi\mu_F^2}{m_q^2} + 4\right]$$

Covariant approach and Covariant projectors

$$\sum_{s_t} u(p_t, s_t)\bar{u}(p_t, s_t) = (\not{p}_t + m_t) \to u\bar{u} = (1 - \gamma_5 \not{s}_t)(\not{p}_t + m_t)/2$$

Completeness relation:

$$\overline{|M|^2} \sim \sum_{\lambda=0,\pm1} \epsilon^{\mu}(\lambda)\epsilon^{\nu\star}(\lambda) \quad , \quad \sum_{\lambda=0,\pm1} \epsilon^{\mu}(\lambda)\epsilon^{\nu\star}(\lambda) = -g^{\mu\nu} + \frac{p_W^{\mu} p_W^{\nu}}{m_W^2}$$

Longitudinal helicity:

$$\epsilon^{\mu}(0)\epsilon^{\nu\star}(0) = \frac{\omega}{|\overrightarrow{P_W}|^2}\left(p_t^{\mu} - \frac{p_t . p_W}{m_W^2} p_W^{\mu}\right)\left(p_t^{\nu} - \frac{p_t . p_W}{m_W^2} p_W^{\nu}\right) \quad \omega = \frac{m_W^2}{m_t^2}$$

Transverse-plus and transverse-minus helicities:

$$\epsilon^{\mu}(\pm)\epsilon^{\nu\star}(\pm) = \frac{1}{2}\Big(- g^{\mu\nu} + \frac{p_W^{\mu} p_W^{\nu}}{m_W^2} - \frac{\omega}{|\overrightarrow{P_W}|^2}\left(p_t^{\mu} - \frac{p_t . p_W}{m_W^2} p_W^{\mu}\right)\left(p_t^{\nu} - \frac{p_t . p_W}{m_W^2} p_W^{\nu}\right) \mp \frac{i\epsilon^{\mu\nu\alpha\beta}}{m_t|\overrightarrow{P_W}|}(p_t)_{\alpha}(p_W)_{\beta}\Big)$$

where: $\epsilon^{0123} = 1$ *and* $|\overrightarrow{P_W}|^2 = (m_t - E_b - E_g)^2 - m_W^2$

Analytical results at NLO

Desired quantity: Angular distribution of differential decay rate

$$\frac{1}{\Gamma_0}\frac{d^2\hat{\Gamma}}{dx_b\ d\cos\theta} = \hat{H}_{++}\cdot\frac{3}{8}(1+\cos\theta)^2 + \hat{H}_{--}\cdot\frac{3}{8}(1-\cos\theta)^2 + \hat{H}_{00}\cdot\frac{3}{4}\sin^2\theta,$$

Radiative corrections to three helicity rates:

$$x_b = E_b/E_b^{max} \quad , \quad \omega = (m_W/m_t)^2$$

$$\hat{H}_{00} = \frac{1}{1+2\omega}\delta(1-x_b) + \frac{\alpha_S C_F}{2\pi(1+2\omega)}B(\mu, x_b) \Longrightarrow \int_0^1 dx_b\ \hat{H}_{00}^{NLO} = 0.635937$$

$$\hat{H}_{--} = \frac{2\omega}{1+2\omega}\delta(1-x_b) + \frac{2\alpha_S \omega C_F}{2\pi(1+2\omega)}C(\mu, x_b) \Longrightarrow \int_0^1 dx_b\ \hat{H}_{00}^{NLO} = 0.277517$$

$$\hat{H}_{++} = \frac{\alpha_S \omega C_F}{2\pi(1+2\omega)}D(\mu, x_b) \Longrightarrow \hat{\Gamma}_{++}^{NLO} = \int_0^1 dx_b\ \hat{H}_{00}^{NLO} = 0.000928$$

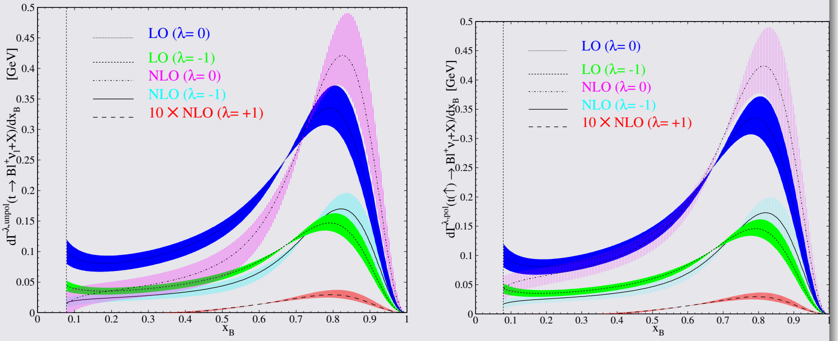
Theoretical and Experimental results from CDF Collaboration

$$\frac{\hat{\Gamma}_{00}}{\hat{\Gamma}} = 0.6955 \quad , \quad \frac{\hat{\Gamma}_{--}}{\hat{\Gamma}} = 0.3035 \quad , \quad \frac{\hat{\Gamma}_{++}}{\hat{\Gamma}} = 0.001$$

$$\frac{\Gamma_{00}^{EXP}}{\Gamma} = 0.91 \pm 0.37(stat) \pm 0.13(syst) \qquad \frac{\Gamma_{++}^{EXP}}{\Gamma} = 0.11 \pm 0.15$$

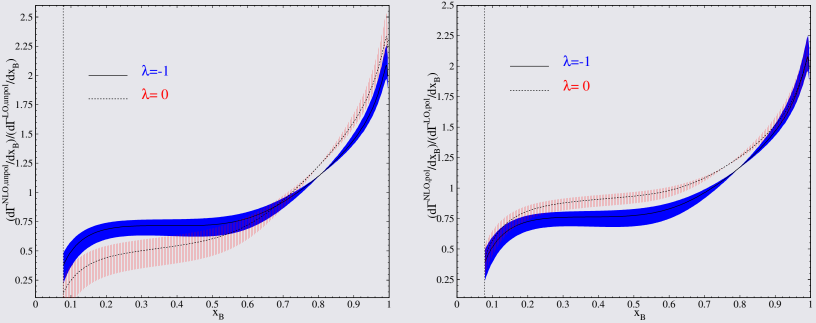
PRD 103 (2021) no.3, 034015 and NPB 862 (2012) 720-736

Our prediction: Energy distribution of B-meson considering helicity contributions of W-boson



Comparison of the NLO contributions of the longitudinal and the transverse-minus and the transverse-plus helicity of the W^+-boson in the B-hadron energy distribution. Here, we set $m_t/2 \leq \mu \leq 2m_t$.

A comparison between LO and NLO results



NLO results for $d\Gamma^{\lambda,\text{unpol}}/dx_B$ and $d\Gamma^{\lambda,\text{pol}}/dx_B$ with $\lambda = 0, -1$, normalized to the respective default LO results, as functions of x_B. The theoretical uncertainties of the NLO results are indicated by the shaded bands.

Top in the theory beyond the Standard Model

Driving most motivations for physics beyond the Standard Model is the fact that the Higgs mass seems unnaturally small.

Hierarchy problem(Renormalized Higgs mass):

$$m_H^2 = m_{0H}^2 + \left(-\frac{3}{8\pi^2}y_t^2\right)\Lambda^2[top] + \left(\frac{9}{64\pi^2}g^2\right)\Lambda^2[vector - bosons] + \left(\frac{1}{64\pi^2}\lambda^2\right)\Lambda^2[Higgs]$$

where Λ is ultraviolet cut-off regulator

when Λ is of order of the GUT scale, cancellations to many digits are required among these contributions.

- Being the main troublemaker, the top may in fact also point to possible new physics in which this problem is avoided
- A popular model is supersymmetry where stop quark loops naturally provide the cancellations of that divergence problems

Top in the theory beyond Standard Model

- In particle physics, a two-Higgs-doublet model (2HDM) is an extension of the Standard Model in which a second Higgs doublet is added to the Higgs sector of the SM
- The addition of the second Higgs doublet, after spontaneous symmetry breaking, leads to five physical states:
  1. The light and heavy CP-even neutral Higgs bosons h and H ($m_H > m_h$)
  2. The CP-odd pseudoscalar Higgs A
  3. Two charged Higgs bosons $H^{\pm}$
- Such a model has six free parameters:
  1. Four Higgs masses ($m_h, m_H, m_A, m_{H^\pm}$)
  2. Ratio of the vacuum expectation values of the two electrically neutral components of the two Higgs doublets ($\tan\beta = < H_2 > / < H_1 >$)
  3. A mixing angle (α)
- Note: No fundamental charged-scalar boson is present in the SM, and the discovery of such a particle would uniquely point to physics beyond the SM.

Searching for new physics through top quarks

- At hadron colliders, for light charged Higgs bosons ($m_{H^\pm} < m_t$) the primary production mechanism is: $t\bar{t} \to H^\pm W^\mp b\bar{b}$ while for heavy Higgs ($m_{H^\pm} > m_t$) associated production of $\bar{t}H^+$ is dominant

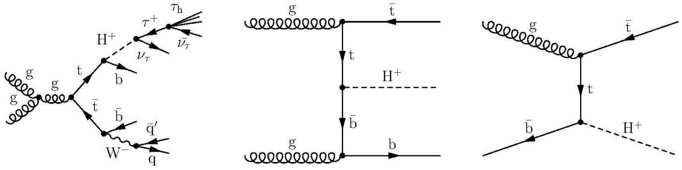


Figure: Example of lowest-order Feynman diagrams for the production of light (left) and heavy (center and right) charged Higgs bosons

The interaction Lagrangian in type I and II 2HDM:

$$L_I = \frac{g_W}{2\sqrt{2}m_W} V_{tb} \cot\beta \Big\{ H^+ \bar{t} \big[m_t(1-\gamma_5) - m_b(1+\gamma_5) \big] b \Big\} + H.c$$

$$L_{II} = \frac{g_W}{2\sqrt{2}m_W} V_{tb} \Big\{ H^+ \bar{t} \big[m_t \cot\beta(1-\gamma_5) + m_b \tan\beta(1+\gamma_5) \big] b \Big\} + H.c.$$

Energy spectrum of B-hadrons in SM and BSM

- For light charged Higgs boson: the total decay width is $\Gamma_t^{tot} = \Gamma_{t\to bW^+}^{SM} + \Gamma_{t\to bH^+}^{2HDM} + \Gamma^{int}$ where $\Gamma_{t\to bW}^{SM} = 1.364$ GeV
- Energy distribution of B-hadron from top decay in the SM and 2HDM

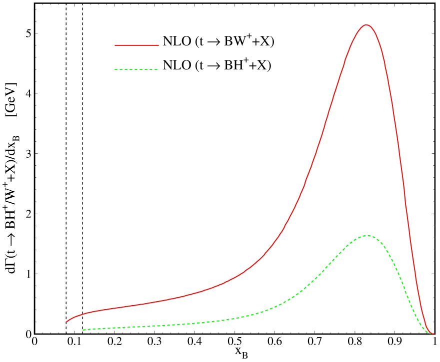
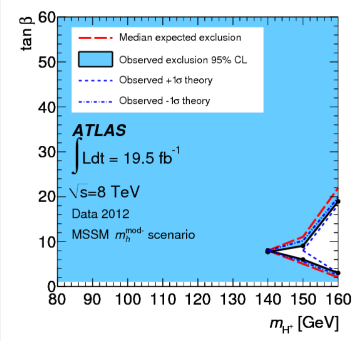
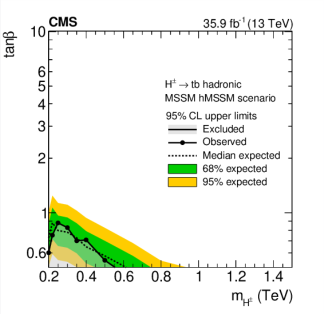


Figure: Our predictions for x_B-spectrum taking $m_H = 100$ GeV and $\tan\beta = 10$

Nucl. Phys. B932 (2018) 505-528.

Results from CERN LHC for light and heavy charged Higgs





Exclusion region in the MSSM $\tan\beta - m_{H^+}$ parameter space for $m_{H^+} = 80 - 1400$ GeV .

MSSM: One of the popular 2HDMs is the minimal supersymmetric SM (MSSM) where one doublet couples to up quarks and the other to down quarks and charged leptons (Type-II 2HDM)

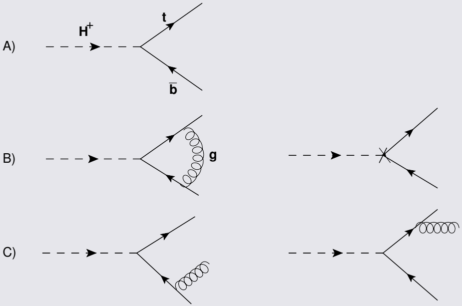
Probing heavy charged Higgs in $H^+ \to t\bar{b}(\to BX)$ channel

In the narrow-width approximation (NWA):

$$\Gamma(H^+ \to b\bar{b}l^+\nu_l) = \Gamma(H^+ \to t\bar{b}) \times Br(t \to bW^+) \times Br(W^+ \to l^+\nu_l)$$

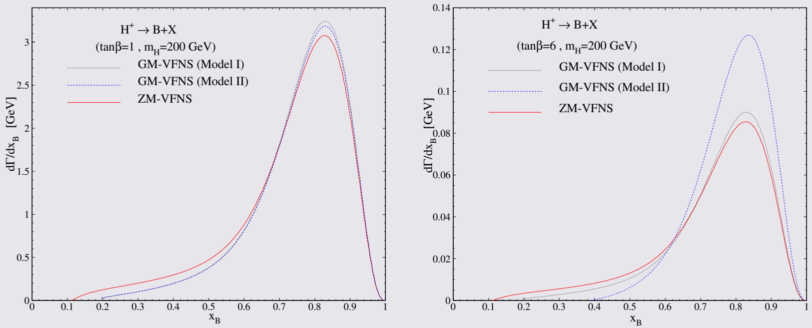
where, $Br(t \to bW^+) = 96.2\%$ and $Br(W^+ \to l^+\nu_l) = 10.86\%$

Radiative Corrections:



$$\frac{d\Gamma}{dx_B}(H^+ \to BX) = \sum \int_{x_B}^{1} \frac{dx_a}{x_a} \frac{d\hat{\Gamma}_a}{dx_a}(\mu_R, \mu_F) D_a^B(\frac{x_B}{x_a}, \mu_F) \quad where \quad x_B = \frac{2E_B}{m_H}$$

B-hadron energy spectrum of Heavy Charged Higgs Boson decay: $H^+ \to t\bar{b}(\to B + jets)$



The x_B spectrum in the decay mode $H^+ \to BX$ at NLO. The GM-VFNs results in two models I and II are compared to the one in the ZM-VFN scheme.

Eur. Phys. J. C (2021) 81:308 and PRD 106, 055040 (2022)

Summary and Conclusion

Top is an object of interest and its attractiveness is due to:

- Its interaction with the Higgs boson
- A tool to search for new physics beyond the SM
- To investigate the mechanism of electroweak symmetry breaking
- $\cdots$

Of particular interest are the distribution in the B-hadron energy in the top quark rest frame, $t \to b(\to B + X) + W^+(\to e^+ + \nu_e)$
LHC will allow for the study of this dominant decay mode

The present work introduces a new channel to indirect search for the charged Higgs boson through the top decay. Any deviation of the total spectrum of bottom-flavored meson from the SM predictions is a reason for the existence of charged Higgs

Last word: Top will remain in the focus of attention for a good many more years.


Thanks for your attention