Fractons

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What is a fracton?

- Fracton phases are characterized by excitations that exhibit restricted mobility, being either immobile (fracton), or mobile only in certain directions(subdimentional particles).

- despite being translationally invariant, our models admit fundamental point-like excitations that are strictly localized in space, and cannot move without paying a finite energy cost to create additional excitations.

- C. Chamon work in 2005
- Haa’s code 2011
- Vijay and Haah 2015

- **Solvable models for gapped fracton phases**
- This approach is limited at present to systems in three spatial dimensions.
- And they gives rise to *gapped* fracton phases.
- The form of the Hamiltonian on a d-dimentional lattice is:

$$H = -\sum \boldsymbol{O}_n \,, \qquad [\boldsymbol{O}_m, \boldsymbol{O}_n] = 0$$

$$\boldsymbol{O}_n{}^2 = 1 \,, \qquad \boldsymbol{O}_n \,|\psi> = \pm 1 \,|\psi>$$

- The operator $\boldsymbol{O}_n$ is required to be a product of an even number of Majorana fermions or pauli matrices.
- This approach has ground state degeneracy D_0.

$$logD_0 = cL^{d-2}$$

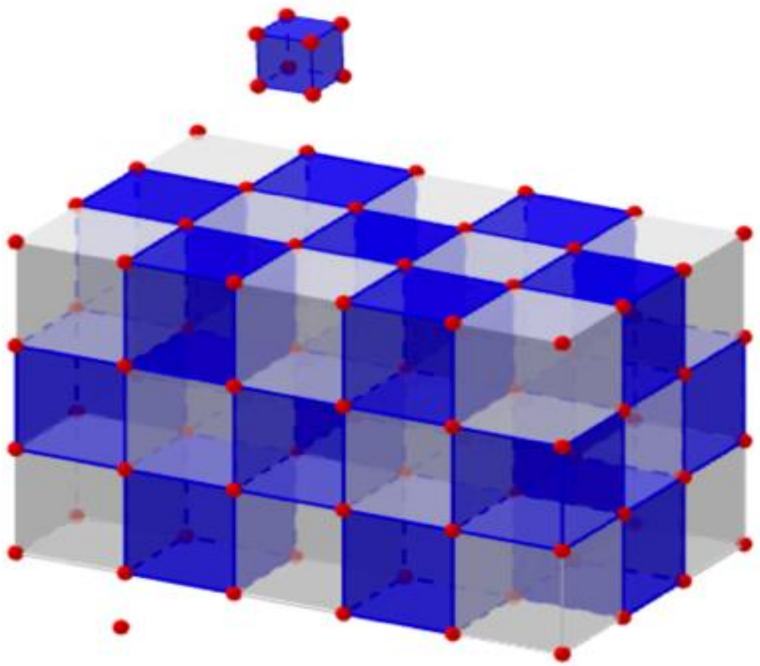
- L is lattice size and c is a constant.

Example:

The Majorana cubic model:

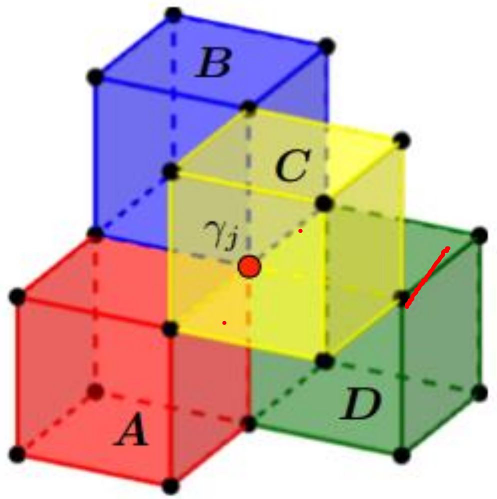
here the $\boldsymbol{O}_n$ operator is the product of the eight Majorana fermions at the vertices of a cube:

$$\boldsymbol{O}_n = \prod_{l \epsilon cube(n)} \gamma_l$$



Acting with a single Majorana operator γ_j creates four excitations.

a) The fundamental cube excitation in this model is completely *immobile*.

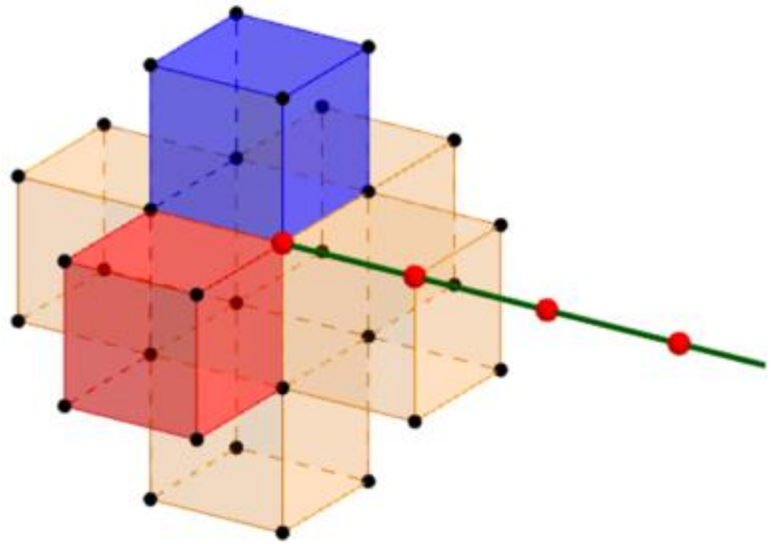


b) **Dimension-1 Particle**

The dimension-1 particle may be created by acting with a single Wilson line operator, defined by the product of the Majorana operators along a straight path l.

$$\widehat{W} \propto \prod_{n \epsilon l} \gamma_n$$

These two-fracton bound-states are only free to move along a line, by simply extending the Wilson line operator.

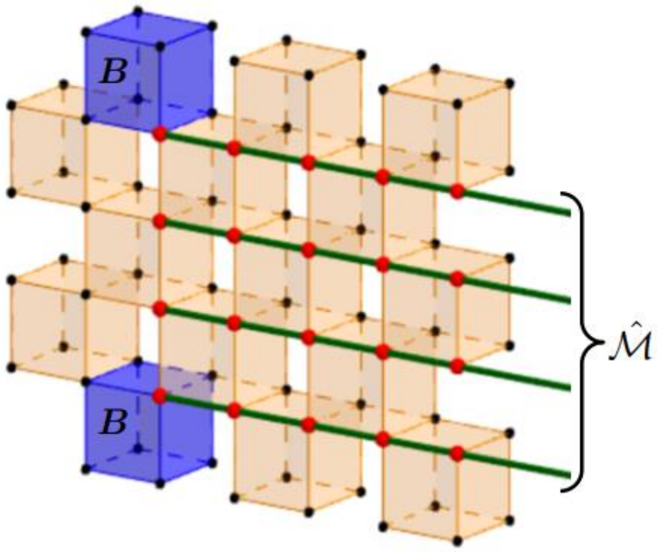


c) **Fractons and Membrane Operators:**

Acting with Majorana operators on a flat, two-dimensional membrane Σ creates fracton excitations at the *corners* of the boundary of Σ.

The membrane operator is:

$$\hat{\Sigma} \propto \prod_{n \epsilon \Sigma} \gamma_n$$



Fracton as an emergent phenomena

- Emergent phenomena:

We refer to a phenomena or physical law as emergent when it arise from the collective behavior of a large number of objects but it’s not manifestly present in the rules governing the individual objects.

A) Emergent higher rank gauge theories

B) Emergent new symmetries

A) Emergent gauge theories

- We use gauge theory to impose constraint.

Example:

$U(1)$ gauge theory:

$$A_i \to A_i + \nabla_i \alpha(x)$$

- First we put the theory on the lattice.
- The gauge field is a vector and must naturally lives on links.
- The electric field E_i is the conjugate of gauge field.

- Now we treat

$$A_i \to A_i + \nabla_i \alpha(x)$$

like a real symmetry and E_i is generator of this symmetry:

$$|\Psi > \to e^{i \int E^i \nabla_i \alpha} \, |\Psi >$$

Integrating by parts

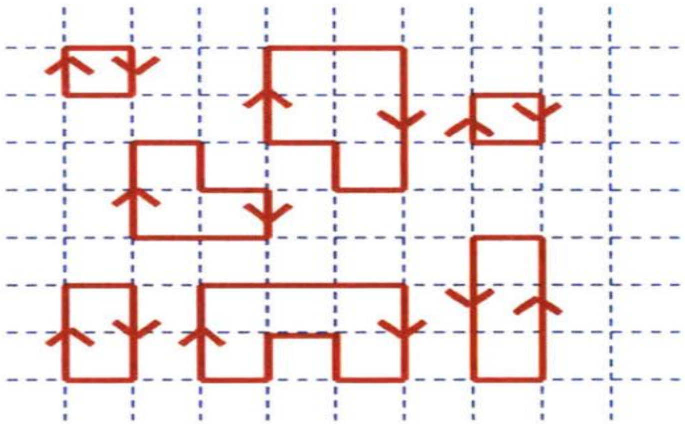
$$|\Psi > \to e^{-i \int \alpha(\nabla_i E^i)} \, |\Psi >$$

We demand that our state be invariant under the gauge transformation

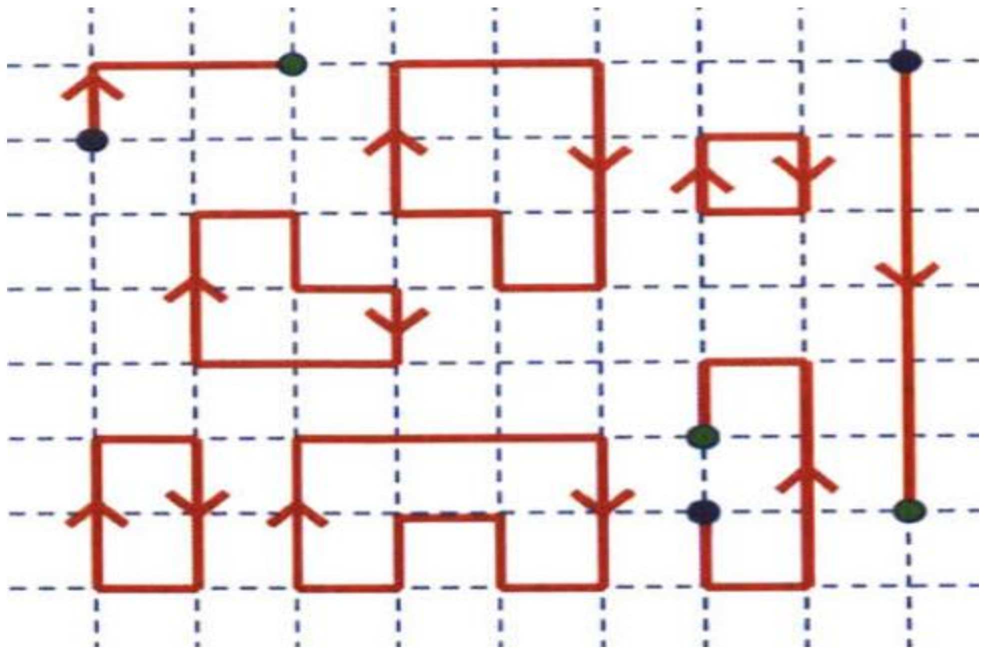
$$e^{-i \int \alpha(\nabla_i E^i)} \, |\Psi> = \, |\Psi>$$

which can only hold if:

$$\nabla_i E^i = 0.$$



- The emergent of quasiparticles when arise $\nabla_i E^i \neq 0$



- we always have two endpoints, indicating that the charges must be created in pairs.
- Now we extend this viewpoint. One could also easily consider other vector gauge fields. For example, a particularly simple gauge theory to work with is the Z_2 gauge theory

Symmetric Tensor Gauge Theories

- **A: scalar charge theory:**

$$A_{ij} \to A_{ij} + \partial_i \partial_j \alpha$$

- Where A_{ij} is a symmetric tensor and E_{ij} is conjugate of this gauge field is also a symmetric tensor.
- The constraint due this symmetry is:

$$\partial_i \partial_j E^{ij} = 0$$

- we therefore write the generalized Gauss's law as:

$$\partial_i \partial_j E^{ij} = \rho$$

- the total charge in any region of our system:

$$\int \rho = \int \partial_i \partial_j E^{ij} = b.t$$

- Net charge cannot be created or destroyed in the bulk of the system.

- This rank 2 theory has an additional conservation law. Let us consider the total dipole moment in some region:

$$\int \vec{x}\rho = \int x^k \partial_i \partial_j E^{jk} = -\int \partial_j E^{jk} + (b.t) = (b.t)$$

- The result is that the total dipole moment is also fixed.

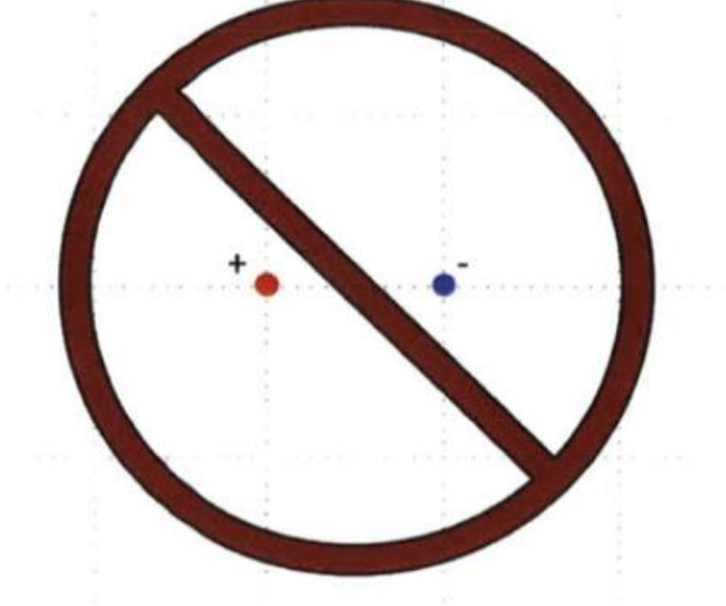
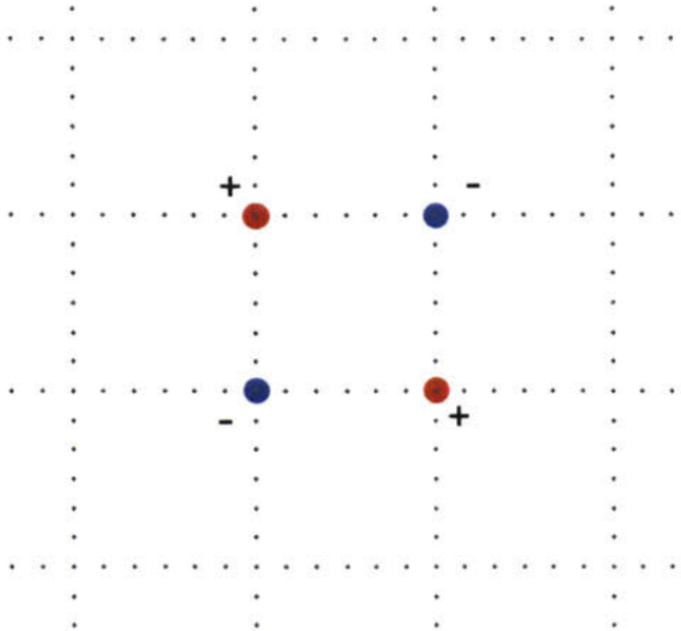
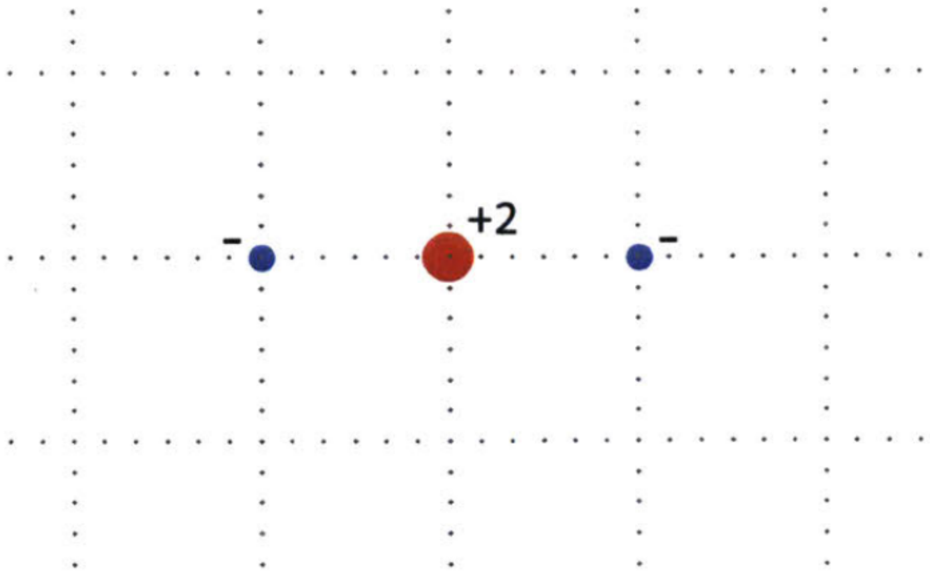


Figure 3-1: A dipole creation operator is disallowed by the dipole conservation law.





- **B. Vector Charge Theory**
- Here gauge transformation is:

$$A_{ij} \rightarrow A_{ij} + \partial_i \theta_j + \partial_j \theta_i$$

The gauge constraint is:

$$\partial_i E^{ij} = 0$$

As in the previous case, we regard the violations of the constraint as the particle states of the theory.

$$\partial_i E^{ij} = \rho^j$$

where the charge ρ^j is now a vector quantity. Even though the charge now has a vector flavor, it still corresponds to point charges:

$$\partial_i \rho^i \neq 0$$

We can also consider the angular moment of this vector charge around an arbitrary origin

$$\int \vec{x} \times \vec{\rho} = \int \epsilon_{ijk} x^j \partial_n E^{nk} = -\int \varepsilon_{ijk} E^{jk} + (b.t) = (b.t)$$

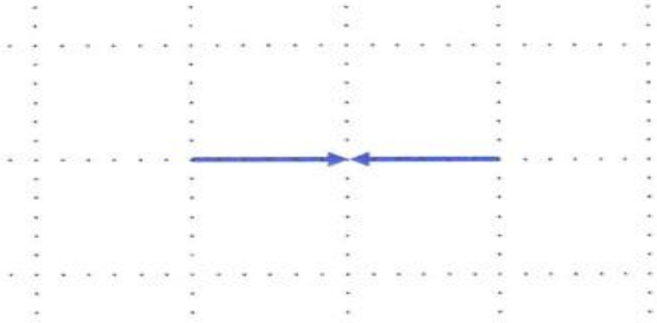


Figure 3-4: Creating head-to-head opposite vectors is consistent with all conservation laws.

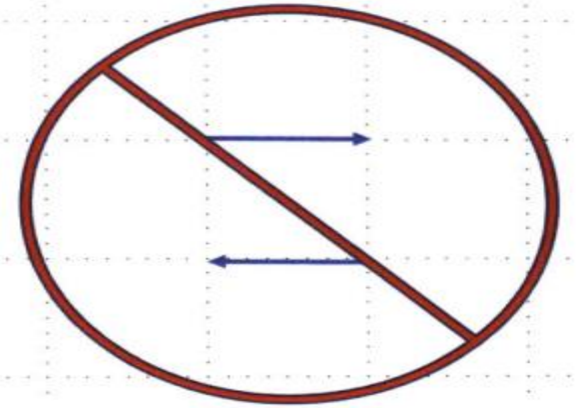
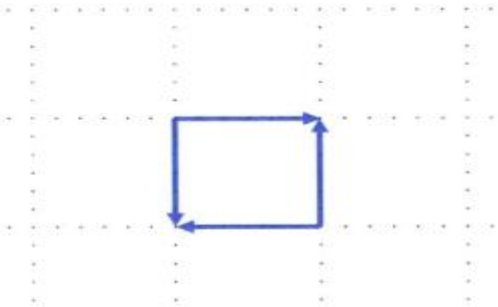


Figure 3-5: Creating side-by-side opposite vectors violates the angular moment conservation law.



- Or we can extend this viewpoint by regarding gauge field as a higher rank tensor:

$$A_{ij} \rightarrow A_{ij} + \partial_i \partial_j \alpha$$

- $The\ conjugate\ of\ A_{ij}$ is E_{ij}. Suppose they are antisymmetric tensors.
- The ground state is $\partial_i E^{ij} = 0$

- On a general state:

$$\partial_i E^{ij} = \rho^j$$

E^{ij} is antisymmetric so we have

$$\partial_i \partial_j E^{ij} = \partial_j \rho^j = 0$$

This means that the charges are actually not point like at all, but rather are forced to exist in closed loop configurations.

B) Emergent new symmetries

In some lattice models if we attempt to write the continuum field theory there will emerge new symmetry that was not in original theory.

For example consider the XY-plaquette model in 2+1 dim:

We have L^x and L^y sites in x and y directions. We lable the site by $s = (\hat{x}, \hat{y})$

$$x = 1,2 \ldots\ldots, L^x$$
$$y = 1,2 \ldots\ldots, L^y$$

we will use $x = a\hat{x}$ and $y = a\hat{y}$ to label the coordinates and $l^x = aL^x$ and $l^y = aL^y$ to denote the physical size of the system.

- The degrees of freedom are phase variable $e^{i\varphi_s}$.
- Their conjugate momenta π_s satisfy

$$[\varphi_s, \pi_{s'}] = i\delta_{ss'}$$

- The Hamiltonian is

$$H = \frac{u}{2}\sum \pi_s^2 - k\sum \cos(\Delta_{xy}\varphi_s)$$

$$\Delta_{xy}\varphi_{\hat{x},\hat{y}} = \varphi_{\hat{x}+1,\hat{y}+1} - \varphi_{\hat{x}+1,\hat{y}} - \varphi_{\hat{x},\hat{y}+1} + \varphi_{\hat{x},\hat{y}}$$

$$\mathcal{L} = \frac{\mu_0}{2}(\partial_0\varphi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\varphi)^2$$

e.o.m
$$\mu_0\partial_0^2\varphi + \frac{1}{\mu}\partial_x^2\partial_y^2\varphi = 0$$

- The shift symmetry on the lattice becomes the subsystem symmetry:

$$\varphi(x,y) \to \varphi(x,y) + f_x(x) + g_y(y)$$

- This warranty the conservation of dipole moment.

- The current of this symmetry is:

$$j_0 = \mu_0 \partial_0 \varphi, \qquad j^{xy} = -\frac{1}{\mu} \partial^x \partial^y \varphi$$

$$\partial_0 j_0 = \partial_x \partial_y j^{xy}$$

The conserved charges of the momentum dipole symmetry are

$$Q_x = \oint dy j_0, \qquad Q_y = \oint dx j_0$$

$$\oint dx Q_x = \oint dy Q_y$$

- Example2:

In some theory if we generalize global u(1) invariance

$$\delta\Phi = i\alpha\Phi \quad \rightarrow \; \delta\Phi = i\vec{\beta}.\vec{x}\Phi$$

The dipole moment is conserve in a complex scalar field theory. This complex scalar theory that describes fracton phases of matter:

$$\mathcal{L} = \dot{\Phi}\dot{\Phi}^* - m^2\Phi^2 - (\partial_i\Phi\partial_j\Phi - \Phi\partial_i\partial_j\Phi)(\partial_i\Phi^*\partial_j\Phi^* - \Phi^*\partial_i\partial_j\Phi^*)$$

Gauging this dipole symmetry requires a purely spatial symmetric tensor gauge field A_{ij} with $\delta A_{ij} = \partial_i\partial_j\Lambda$.

Interacting Fractons in 2+1-Dimensional Quantum Field Theory

- Fracton lagrangian is

$$\boldsymbol{\mathcal{L}} = \frac{\mu_0}{2}(\partial_0\varphi)^2 - \frac{1}{2\mu}(\partial_x\partial_y\varphi)^2$$

e.o.m

$$\mu_0\partial_0^2\varphi + \frac{1}{\mu}\partial_x^2\partial_y^2\varphi = 0$$

- The fermion lagrangian is:

$$\mathcal{L}_{fer} = \psi^\dagger(i\partial_0 + \frac{\nabla^2}{2m} - \gamma)\psi$$

- The simplest local interaction

$$\mathcal{L}_{int} = \lambda\psi^\dagger\psi\partial_x\partial_y\Phi$$

- The scaling dimention of $[\lambda] = 0$ so the theory is renormalisble in 2D

- free fermionic propagator and free propagator for scalar field is:

$$< 0 \ |T\psi^{\dagger}\psi \ |0 >= \frac{1}{\omega - \frac{{k_x}^2 + {k_y}^2}{2m} + \gamma - i\epsilon}$$

$$< 0 \ |T\Phi\Phi \ |0 >= \frac{1}{-\mu_0\omega^2 + \frac{{k_x}^2 {k_y}^2}{\mu} - i \in}$$

The vertex function is:

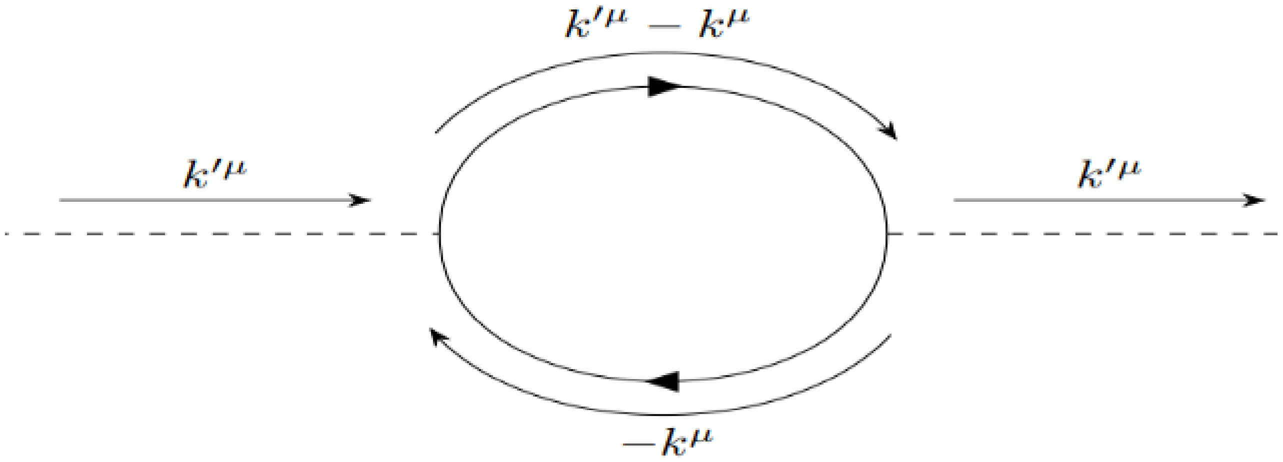
$$Vertex = i\lambda k_x k_y$$

- To renormalize this theory, we introduce local counterterms and subtract the UV divergences using a rotationally-invariant hard momentum cutoff

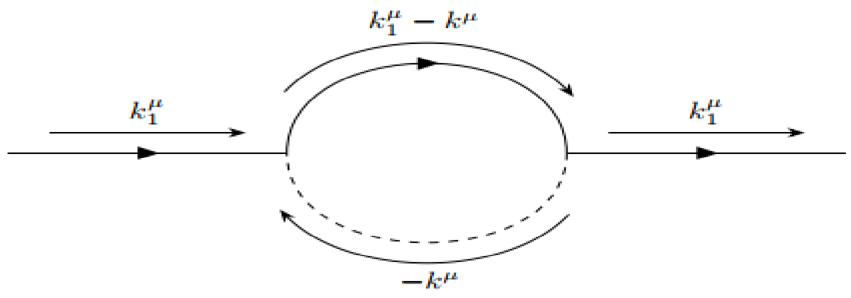
$$\mathcal{L}_c = \frac{\delta_{\mu_0}}{2}(\partial_0\ \Phi)^2 + \frac{\delta_{1/\mu}}{2\mu}(\partial_x\partial_y\Phi)^2 + \delta_{z_\psi}\psi^\dagger(\partial_0)\psi - \frac{\delta_{1/m}}{2}\psi^\dagger\nabla^2\psi - \delta_\lambda\psi^\dagger\psi\partial_x\partial_y\Phi + \delta_\lambda\psi^\dagger\psi$$

- from the Scalar self energy and with a momentum cutoff Λ, The ultraviolet divergenceis absorbed into the $\delta_{1/\mu}$counter-term The exact form of the counter-term is:

$$\delta_{1/\mu} = -\frac{m\lambda^2}{2\pi}\log(\Lambda)$$



- And from the fermionic self energy:



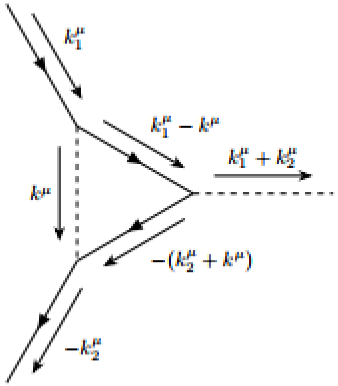
$$\delta_{z_\psi} = -\Omega_1 \log(\Lambda)$$

$$\delta_{1/m} = -\frac{\Omega_1}{m} \log(\Lambda) - \Omega_2 \log(\Lambda)$$

$$\delta_\gamma = -\frac{\Lambda^2}{2} \Omega_3 - \gamma \Omega_1 \log(\Lambda)$$

Where Ω_1, Ω_2, and Ω_3 are three functions of the couplings which are momentum and cutoff independent

And finally form the vertex correction



$$\delta_\lambda = \lambda^3 \frac{m^2 \mu}{\pi^2} \frac{\sqrt{\mu\mu_0}}{\mu\mu_0 - m^2} \left\{ 1 + \frac{m}{(\mu\mu_0 - m^2)^{1/2}} \left(\arctan\left(\frac{m}{\sqrt{\mu\mu_0 - m^2}} \right) - \frac{\pi}{2} \right) \right\} \log(\Lambda)$$

The 1-loop beta function for the dimentionless couplings are:

$$\beta(\lambda) = 0 \qquad \beta({}^{1}/_{m}) = \Omega_2$$

$$\beta({}^{1}/_{\mu}) = \frac{m\lambda^2}{2\pi}$$

Conclusion and outlook