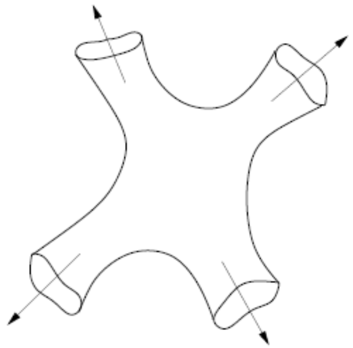
کنش موثر نظریه ریسمان بوزونی (و هتراتیک)

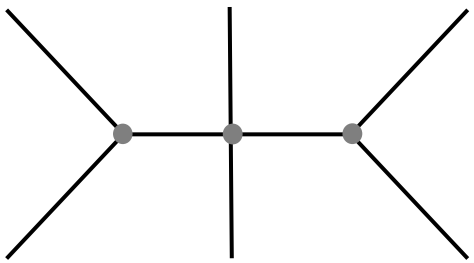
در مرتبه α'^2

Thesis

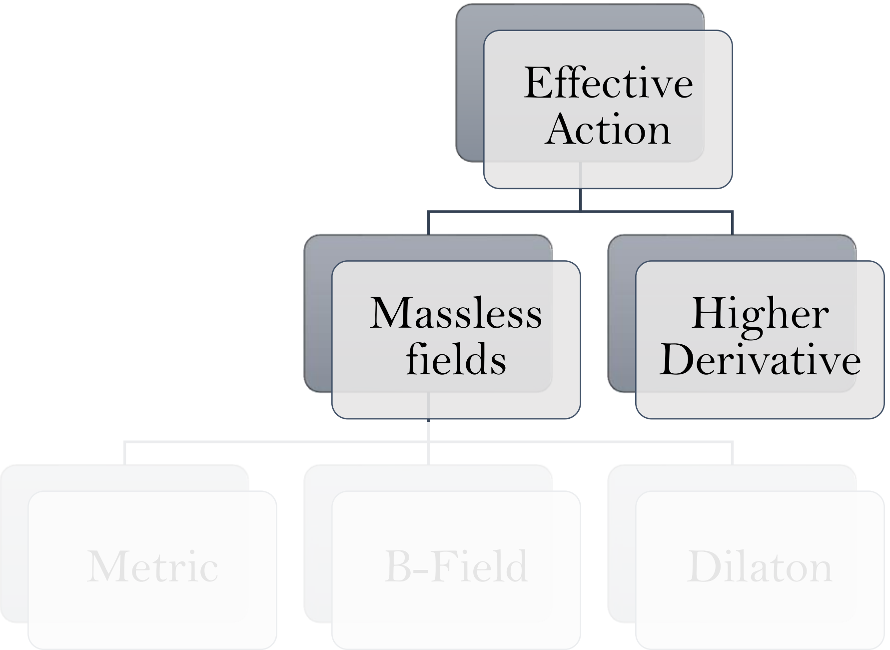
- Couplings at order α'^2
- String scattering
- Feynman diagrams

$$S_n = \int d^Dx\sqrt{-g}e^{-2\Phi}\mathcal{L}_n$$





Effective Action



$$S_{\text{eff}} = \sum_{n=0}^{\infty} \alpha'^n S_n = S_0 + \alpha' S_1 + \alpha'^2 S_2 + \cdots \; ; \quad S_n = \int d^D x \sqrt{-g} e^{-2\Phi} \mathcal{L}_n$$

Couplings

R, H, Φ
The most general gauge inv. Lagrangian
Couplings are not independent
α' order
41
8
4
α'^2 order
705
60
27

Couplings

Primary Action + Total Derivative + Field Redefinition + Bianchi Identities = Secondary Action

α' Order

Field Redefinitions

$$\begin{aligned} g_{\mu\nu} &\rightarrow g_{\mu\nu} + \alpha' \delta g^{(1)}_{\mu\nu} \\ B_{\mu\nu} &\rightarrow B_{\mu\nu} + \alpha' \delta B^{(1)}_{\mu\nu} \\ \Phi &\rightarrow \Phi + \alpha' \delta \Phi^{(1)} \end{aligned}$$

$$\begin{aligned} \delta g^{(1)}_{\mu\nu} =& a_1 R_{\mu\nu} + a_2 H_\mu{}^{\alpha\beta} H_{\nu\alpha\beta} + a_3 \nabla_\nu \nabla_\mu \Phi + a_4 \nabla_\mu \Phi \nabla_\nu \Phi + g_{\mu\nu} \Big(a_5 R + a_6 H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \\ & + a_7 \nabla_\alpha \nabla^\alpha \Phi + a_8 \nabla_\alpha \Phi \nabla^\alpha \Phi \Big) \\ \delta B^{(1)}_{\mu\nu} =& a_9 \nabla_\alpha H_{\mu\nu}{}^\alpha + a_{10} H_{\mu\nu\alpha} \nabla^\alpha \Phi \\ \delta \Phi^{(1)} =& a_{11} R + a_{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} + a_{13} \nabla_\alpha \nabla^\alpha \Phi + a_{14} \nabla_\alpha \Phi \nabla^\alpha \Phi \end{aligned}$$

$$\delta S_0 = \frac{\delta S_0}{\delta g_{\alpha\beta}} \delta g^{(1)}_{\alpha\beta} + \frac{\delta S_0}{\delta B_{\alpha\beta}} \delta B^{(1)}_{\alpha\beta} + \frac{\delta S_0}{\delta \Phi} \delta \Phi^{(1)} \equiv \int d^D x \sqrt{-g} e^{-2\Phi} \mathcal{K}_1$$

α' Order

$$
\begin{aligned}
\mathbf{S}^{(1)}_{MT} = \frac{-2\alpha' a_1}{\kappa^2}\int d^D x\, e^{-2\phi}\sqrt{-G}\Big(& R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - \frac{1}{2}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon} \\
& + \frac{1}{24}H_{\epsilon\delta\zeta}H^{\epsilon}{}_{\alpha}{}^{\beta}H^{\delta}{}_{\beta}{}^{\gamma}H^{\zeta}{}_{\gamma}{}^{\alpha} - \frac{1}{8}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\zeta}H_{\delta\epsilon\zeta}\Big)
\end{aligned}
$$

Metsaev-Tseytlin

$$
\begin{aligned}
\mathrm{S}^{(1)}_{M} = -\frac{2\alpha' a_1}{\kappa^2}\int d^D x\sqrt{-G}e^{-2\Phi}\Big[& R^2_{GB} + \frac{1}{24}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\varepsilon}H_{\gamma\epsilon\varepsilon} - \frac{1}{8}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon} \\
& + \frac{1}{144}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}H_{\delta\epsilon\varepsilon}H^{\delta\epsilon\varepsilon} + H_{\alpha}{}^{\gamma\delta}H_{\beta\gamma\delta}R^{\alpha\beta} - \frac{1}{6}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}R - \frac{1}{2}H_{\alpha}{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon} \\
& - \frac{2}{3}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\nabla^\alpha\Phi + \frac{2}{3}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi + 8R\nabla_\alpha\Phi\nabla^\alpha\Phi - 16R_{\alpha\beta}\nabla^\alpha\Phi\nabla^\beta\Phi \\
& + 16\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\nabla^\beta\Phi - 16\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + 2H_{\alpha}{}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^\beta\nabla^\alpha\Phi\Big],
\end{aligned}
$$

Meissner

$$R^2_{GB} = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} - 4R^{\alpha\beta}R_{\alpha\beta} + R^2$$

α'^2 Order

$$
\begin{aligned}
S^{(2)B}_{MT} = {} & \frac{-2\alpha'^2 a_1^2}{\kappa^2}\int d^{26}x\, e^{-2\Phi}\sqrt{-G}\Big[-\frac{1}{12}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\zeta}H_\gamma{}^{\iota\kappa}H_{\epsilon\iota}{}^{\mu}H_{\zeta\kappa\mu} \\
& +\frac{1}{30}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\delta{}^{\iota\kappa}H_{\epsilon\zeta}{}^{\mu}H_{\iota\kappa\mu}+\frac{3}{10}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_{\delta\epsilon}{}^{\iota}H_\zeta{}^{\kappa\mu}H_{\iota\kappa\mu} \\
& +\frac{13}{20}H_\alpha{}^{\epsilon\zeta}H_\beta{}^{\iota\kappa}H_{\gamma\epsilon\zeta}H_{\delta\iota\kappa}R^{\alpha\beta\gamma\delta}+\frac{2}{5}H_\alpha{}^{\epsilon\zeta}H_{\beta\epsilon}{}^{\iota}H_{\gamma\zeta}{}^{\kappa}H_{\delta\iota\kappa}R^{\alpha\beta\gamma\delta}+\frac{18}{5}H_{\alpha\gamma}{}^{\epsilon}H_\beta{}^{\zeta\iota}H_{\delta\zeta}{}^{\kappa}H_{\epsilon\iota\kappa}R^{\alpha\beta\gamma\delta} \\
& -\frac{43}{5}H_{\alpha\gamma}{}^{\epsilon}H_\beta{}^{\zeta\iota}H_{\delta\epsilon}{}^{\kappa}H_{\zeta\iota\kappa}R^{\alpha\beta\gamma\delta}-\frac{16}{5}H_{\alpha\gamma}{}^{\epsilon}H_{\beta\delta}{}^{\zeta}H_\epsilon{}^{\iota\kappa}H_{\zeta\iota\kappa}R^{\alpha\beta\gamma\delta}-2H_{\beta\epsilon}{}^{\iota}H_{\delta\zeta\iota}R_\alpha{}^{\epsilon}{}_\gamma{}^{\zeta}R^{\alpha\beta\gamma\delta} \\
& -2H_{\beta\delta}{}^{\iota}H_{\epsilon\zeta\iota}R_\alpha{}^{\epsilon}{}_\gamma{}^{\zeta}R^{\alpha\beta\gamma\delta}-\frac{4}{3}R_\alpha{}^{\epsilon}{}_\gamma{}^{\zeta}R^{\alpha\beta\gamma\delta}R_{\beta\zeta\delta\epsilon}+\frac{4}{3}R_{\alpha\beta}{}^{\epsilon\zeta}R^{\alpha\beta\gamma\delta}R_{\gamma\epsilon\delta\zeta}+3H_\beta{}^{\zeta\iota}H_{\epsilon\zeta\iota}R^{\alpha\beta\gamma\delta}R_\gamma{}^{\epsilon}{}_{\alpha\delta} \\
& +2H_{\beta\epsilon}{}^{\iota}H_{\delta\zeta\iota}R^{\alpha\beta\gamma\delta}R_\gamma{}^{\epsilon}{}_\alpha{}^{\zeta}+2H_{\alpha\beta\epsilon}H_{\delta\zeta\iota}R^{\alpha\beta\gamma\delta}R_\gamma{}^{\epsilon\zeta\iota}+\frac{13}{10}H_\alpha{}^{\gamma\delta}H_{\beta\gamma}{}^{\epsilon}H_\delta{}^{\zeta\iota}H_{\epsilon\zeta\iota}\nabla^\beta\nabla^\alpha\Phi \\
& +\frac{13}{5}H_\gamma{}^{\epsilon\zeta}H_{\delta\epsilon\zeta}R_\alpha{}^{\gamma}{}_\beta{}^{\delta}\nabla^\beta\nabla^\alpha\Phi-\frac{52}{5}H_{\beta\delta}{}^{\zeta}H_{\gamma\epsilon\zeta}R_\alpha{}^{\gamma\delta\epsilon}\nabla^\beta\nabla^\alpha\Phi-\frac{26}{5}H_{\alpha\gamma\epsilon}H_{\beta\delta\zeta}R^{\gamma\delta\epsilon\zeta}\nabla^\beta\nabla^\alpha\Phi \\
& +\frac{13}{5}\nabla^\beta\nabla^\alpha\Phi\nabla_\epsilon H_{\beta\gamma\delta}\nabla^\epsilon H_\alpha{}^{\gamma\delta}+\frac{13}{10}H_{\beta\gamma}{}^{\epsilon}H^{\beta\gamma\delta}H_\delta{}^{\zeta\iota}\nabla^\alpha\Phi\nabla_\iota H_{\alpha\epsilon\zeta}+\frac{1}{20}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_{\beta\gamma}{}^{\zeta} \\
& -\frac{13}{20}H_\alpha{}^{\beta\gamma}H_{\delta\epsilon}{}^{\iota}H^{\delta\epsilon\zeta}\nabla^\alpha\Phi\nabla_\iota H_{\beta\gamma\zeta}+\frac{1}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\zeta H_{\gamma\epsilon\iota}\nabla^\iota H_{\beta\delta}{}^{\zeta} \\
& -\frac{6}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\gamma\epsilon\zeta}\nabla^\iota H_{\beta\delta}{}^{\zeta}-\frac{6}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\zeta H_{\delta\epsilon\iota}\nabla^\iota H_\gamma{}^{\epsilon\zeta}+\frac{17}{10}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_\gamma{}^{\epsilon\zeta}\Big],
\end{aligned}
$$

$$
\begin{aligned}
S^{(2)B}_{M} = {} & \frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{26}x\sqrt{-G}e^{-2\Phi}\Big[\frac{1}{12}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\zeta}H_\gamma{}^{\iota\kappa}H_{\epsilon\iota}{}^{\mu}H_{\zeta\kappa\mu} \\
& -\frac{1}{30}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\delta{}^{\iota\kappa}H_{\epsilon\zeta}{}^{\mu}H_{\iota\kappa\mu}-\frac{1}{20}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_{\delta\epsilon}{}^{\iota}H_\zeta{}^{\kappa\mu}H_{\iota\kappa\mu} \\
& +\frac{4}{3}R_\alpha{}^{\epsilon}{}_\gamma{}^{\zeta}R^{\alpha\beta\gamma\delta}R_{\beta\zeta\delta\epsilon}-\frac{4}{3}R_{\alpha\beta}{}^{\epsilon\zeta}R^{\alpha\beta\gamma\delta}R_{\gamma\epsilon\delta\zeta}-\frac{2}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_\beta{}^{\zeta\iota}H_{\delta\zeta}{}^{\kappa}R_{\gamma\epsilon\iota\kappa} \\
& +2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\delta{}^{\iota}R_{\gamma\zeta\epsilon\iota}-\frac{3}{20}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\epsilon\zeta}{}^{\kappa}H^{\epsilon\zeta\iota}R_{\gamma\iota\delta\kappa}-2H^{\alpha\beta\gamma}H^{\delta\epsilon\zeta}R_{\alpha\beta\delta}{}^{\iota}R_{\gamma\iota\epsilon\zeta} \\
& -2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\delta{}^{\iota}R_{\gamma\iota\epsilon\zeta}+2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\gamma{}^{\iota}R_{\delta\zeta\epsilon\iota}+H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}R_\gamma{}^{\epsilon\zeta\iota}R_{\delta\zeta\epsilon\iota} \\
& -\frac{3}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\epsilon{}^{\iota\kappa}R_{\delta\iota\zeta\kappa}-\frac{8}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\zeta}H_\gamma{}^{\iota\kappa}R_{\epsilon\iota\zeta\kappa}+\frac{1}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\delta{}^{\iota\kappa}R_{\epsilon\iota\zeta\kappa} \\
& -\frac{3}{10}H_\alpha{}^{\gamma\delta}H_{\beta\gamma}{}^{\epsilon}H_\delta{}^{\zeta\iota}H_{\epsilon\zeta\iota}\nabla^\beta\nabla^\alpha\Phi-\frac{3}{5}H_{\gamma\delta}{}^{\zeta}H^{\gamma\delta\epsilon}R_{\alpha\epsilon\beta\zeta}\nabla^\beta\nabla^\alpha\Phi-\frac{12}{5}H_\alpha{}^{\gamma\delta}H_\gamma{}^{\epsilon\zeta}R_{\beta\epsilon\delta\zeta}\nabla^\beta\nabla^\alpha\Phi \\
& +\frac{6}{5}H_\alpha{}^{\gamma\delta}H_\beta{}^{\epsilon\zeta}R_{\gamma\epsilon\delta\zeta}\nabla^\beta\nabla^\alpha\Phi-\frac{3}{5}\nabla^\beta\nabla^\alpha\Phi\nabla_\epsilon H_{\beta\gamma\delta}\nabla^\epsilon H_\alpha{}^{\gamma\delta}-\frac{3}{10}H_{\beta\gamma}{}^{\epsilon}H^{\beta\gamma\delta}H_\delta{}^{\zeta\iota}\nabla^\alpha\Phi\nabla_\iota H_{\alpha\epsilon\zeta} \\
& +\frac{3}{20}H_\alpha{}^{\beta\gamma}H_{\delta\epsilon}{}^{\iota}H^{\delta\epsilon\zeta}\nabla^\alpha\Phi\nabla_\iota H_{\beta\gamma\zeta}-\frac{1}{20}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_{\beta\gamma}{}^{\zeta}-\frac{1}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\zeta H_{\gamma\epsilon\iota}\nabla^\iota H_{\beta\delta}{}^{\zeta} \\
& +\frac{1}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\gamma\epsilon\zeta}\nabla^\iota H_{\beta\delta}{}^{\zeta}+\frac{1}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\zeta H_{\delta\epsilon\iota}\nabla^\iota H_\gamma{}^{\epsilon\zeta}-\frac{1}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_\gamma{}^{\epsilon\zeta}\Big].
\end{aligned}
$$

α'^2 Order

$$
\begin{aligned}
g_{\mu\nu} &\rightarrow g_{\mu\nu} + \alpha'\delta g^{(1)}_{\mu\nu} + \alpha'^2\delta g^{(2)}_{\mu\nu} \\
B_{\mu\nu} &\rightarrow B_{\mu\nu} + \alpha'\delta B^{(1)}_{\mu\nu} + \alpha'^2\delta B^{(2)}_{\mu\nu} \\
\Phi &\rightarrow \Phi + \alpha'\delta\Phi^{(1)} + \alpha'^2\delta\Phi^{(2)}
\end{aligned}
$$

$$
\begin{aligned}
\delta S_0 + \delta S_1 = {} & \boxed{\frac{\delta S_0}{\delta g_{\alpha\beta}}\delta g^{(2)}_{\alpha\beta} + \frac{\delta S_0}{\delta B_{\alpha\beta}}\delta B^{(2)}_{\alpha\beta} + \frac{\delta S_0}{\delta \Phi}\delta\Phi^{(2)}} + \frac{\delta S_1}{\delta g_{\alpha\beta}}\delta g^{(1)}_{\alpha\beta} + \frac{\delta S_1}{\delta B_{\alpha\beta}}\delta B^{(1)}_{\alpha\beta} + \frac{\delta S_1}{\delta \Phi}\delta\Phi^{(1)} \\
& + S_0(\delta g^{(1)}, \delta g^{(1)}) + S_0(\delta g^{(1)}, \delta B^{(1)}) + S_0(\delta g^{(1)}, \delta\Phi^{(1)}) \\
& + S_0(\delta B^{(1)}, \delta B^{(1)}) + S_0(\delta B^{(1)}, \delta\Phi^{(1)}) + S_0(\delta\Phi^{(1)}, \delta\Phi^{(1)})
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{S}_M^{(2)B} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{26}x\sqrt{-G}e^{-2\Phi}\Big[& -\frac{4}{3}R_{\alpha}{}^{\kappa}{}_{\gamma}{}^{\lambda}R^{\alpha\beta\gamma\theta}R_{\beta\lambda\theta\kappa} + \frac{4}{3}R_{\alpha\beta}{}^{\kappa\lambda}R^{\alpha\beta\gamma\theta}R_{\gamma\kappa\theta\lambda} - \frac{1}{12}H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}H_{\beta\theta}{}^{\lambda}H_{\gamma}{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau} \\
& + \frac{1}{4}H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\kappa\lambda}H_{\theta}{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau} + \frac{1}{48}H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\kappa\lambda}H_{\theta}{}^{\mu\nu}H_{\kappa\lambda}{}^{\tau}H_{\mu\nu\tau} - 2H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta\theta}{}^{\lambda\mu}R_{\gamma\lambda\kappa\mu} - H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}R_{\gamma}{}^{\kappa\lambda\mu}R_{\theta\lambda\kappa\mu} \\
& + 2H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}R_{\alpha\beta\theta}{}^{\mu}R_{\gamma\mu\kappa\lambda} - 2H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta}{}^{\lambda}{}_{\gamma}{}^{\mu}R_{\theta\lambda\kappa\mu} + \frac{1}{4}H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}\nabla_{\gamma}H_{\kappa\lambda\mu}\nabla_{\theta}H_{\alpha\beta}{}^{\mu} + \frac{1}{2}H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_{\kappa}H_{\theta\lambda\mu}\nabla^{\mu}H_{\beta\gamma}{}^{\lambda} \\
& + H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_{\mu}H_{\gamma\kappa\lambda}\nabla^{\mu}H_{\beta\theta}{}^{\lambda}\Big].
\end{aligned}
$$

H. Gholian and M. R. Garousi, Phys. Rev. D **109**, no.8, 086007 (2024) doi:10.1103/PhysRevD.109.086007 [arXiv:2311.05207 [hep-th]].

Heterotic

$$H_{\mu\nu\alpha} \rightarrow H_{\mu\nu\alpha} + \tfrac{3}{2}\alpha'\Omega_{\mu\nu\alpha}$$

$$\mathbf{S}^{(0)} = -\frac{2}{\kappa^2}\int d^D x\, e^{-2\phi}\sqrt{-G}\left(R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H^2\right)$$

$$\mathbf{S}_O^{(1)} = -\frac{2\alpha' a_1}{\kappa^2}\int d^{10}x\sqrt{-G}\,e^{-2\Phi}\left(-2H_{\mu\nu\alpha}\Omega^{\mu\nu\alpha}\right)$$ T-duality

$$\mathbf{S}_{1e}^{(2)} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{10}x\sqrt{-G}\,e^{-2\Phi}\left(-12\Omega_{\mu\nu\alpha}\Omega^{\mu\nu\alpha}\right)$$ T-duality

$$\begin{aligned}\mathbf{S}_{1O}^{(2)} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\Big[&-4H^{\alpha\beta\gamma}R\Omega_{\alpha\beta\gamma} - 12H^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon}\Omega_\alpha{}^{\delta\epsilon} + 24H_\alpha{}^{\gamma\delta}R^{\alpha\beta}\Omega_{\beta\gamma\delta}\\ &+2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\varepsilon}\Omega_{\gamma\epsilon\varepsilon} - 6H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\varepsilon}\Omega_{\delta\epsilon\varepsilon} + \frac{1}{3}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}H^{\delta\epsilon\varepsilon}\Omega_{\delta\epsilon\varepsilon}\\ &-16H^{\beta\gamma\delta}\Omega_{\beta\gamma\delta}\nabla_\alpha\nabla^\alpha\Phi + 16H^{\beta\gamma\delta}\Omega_{\beta\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi + 48H_\alpha{}^{\gamma\delta}\Omega_{\beta\gamma\delta}\nabla^\beta\nabla^\alpha\Phi\Big]\end{aligned}$$

T-duality

$$\begin{aligned}\mathbf{S}_{2e}^{(2)} = \frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\Big[&\frac{1}{12}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\zeta}H_\gamma{}^{\iota\kappa}H_{\epsilon\iota}{}^{\mu}H_{\zeta\kappa\mu} - \frac{1}{80}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\delta{}^{\iota\kappa}H_{\epsilon\zeta}{}^{\mu}H_{\iota\kappa\mu}\\ &+\frac{1}{80}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_{\delta\epsilon}{}^{\iota}H_\zeta{}^{\kappa\mu}H_{\iota\kappa\mu} - \frac{2}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_\beta{}^{\zeta\iota}H_{\delta\zeta}{}^{\kappa}R_{\gamma\epsilon\iota\kappa} + 2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\delta{}^{\iota}R_{\gamma\zeta\epsilon\iota} - 2H^{\alpha\beta\gamma}H^{\delta\epsilon\zeta}R_{\alpha\beta\delta}{}^{\iota}R_{\gamma\iota\epsilon\zeta}\\ &-\frac{1}{40}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_{\epsilon\zeta}{}^{\kappa}H^{\epsilon\zeta\iota}R_{\gamma\iota\delta\kappa} - 2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\delta{}^{\iota}R_{\gamma\iota\epsilon\zeta} + 2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_\beta{}^{\zeta}{}_\gamma{}^{\iota}R_{\delta\zeta\epsilon\iota} - \frac{1}{10}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\epsilon{}^{\iota\kappa}R_{\delta\iota\zeta\kappa}\\ &-\frac{8}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\zeta}H_\gamma{}^{\iota\kappa}R_{\epsilon\iota\zeta\kappa} - \frac{1}{20}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\zeta}H_\delta{}^{\iota\kappa}R_{\epsilon\iota\zeta\kappa} - \frac{1}{20}H_\alpha{}^{\gamma\delta}H_{\beta\gamma}{}^{\epsilon}H_\delta{}^{\zeta\iota}H_{\epsilon\zeta\iota}\nabla^\beta\nabla^\alpha\Phi - \frac{1}{10}H_{\gamma\delta}{}^{\zeta}H^{\gamma\delta\epsilon}R_{\alpha\epsilon\beta\zeta}\nabla^\beta\nabla^\alpha\Phi\\ &-\frac{2}{5}H_\alpha{}^{\gamma\delta}H_\gamma{}^{\epsilon\zeta}R_{\beta\epsilon\delta\zeta}\nabla^\beta\nabla^\alpha\Phi + \frac{1}{5}H_\alpha{}^{\gamma\delta}H_\beta{}^{\epsilon\zeta}R_{\gamma\epsilon\delta\zeta}\nabla^\beta\nabla^\alpha\Phi - \frac{1}{10}\nabla^\beta\nabla^\alpha\Phi\nabla_\epsilon H_{\beta\gamma\delta}\nabla^\epsilon H_\alpha{}^{\gamma\delta} - \frac{1}{20}H_{\beta\gamma}{}^{\epsilon}H^{\beta\gamma\delta}H_\delta{}^{\zeta\iota}\nabla^\alpha\Phi\nabla_\iota H_{\alpha\epsilon\zeta}\\ &+\frac{1}{40}H_\alpha{}^{\beta\gamma}H_{\delta\epsilon}{}^{\iota}H^{\delta\epsilon\zeta}\nabla^\alpha\Phi\nabla_\iota H_{\beta\gamma\zeta} - \frac{1}{20}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_{\beta\gamma}{}^{\zeta} - \frac{1}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\zeta H_{\gamma\epsilon\iota}\nabla^\iota H_{\beta\delta}{}^{\zeta} + \frac{1}{5}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}\nabla_\iota H_{\gamma\epsilon\zeta}\nabla^\iota H_{\beta\delta}{}^{\zeta}\\ &+\frac{1}{5}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\zeta H_{\delta\epsilon\iota}\nabla^\iota H_\gamma{}^{\epsilon\zeta} - \frac{3}{40}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}\nabla_\iota H_{\delta\epsilon\zeta}\nabla^\iota H_\gamma{}^{\epsilon\zeta}\Big].\end{aligned}$$

$$\begin{aligned}\mathbf{S}_{2O}^{(2)} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\Big[&4H^{\alpha\beta\gamma}H^{\delta\epsilon\varepsilon}R_{\gamma\epsilon\varepsilon\mu}\nabla_\beta H_{\alpha\delta}{}^{\mu} - 2H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\delta\epsilon\varepsilon\mu}\nabla^\mu H_{\beta\gamma}{}^{\varepsilon}\\ &-\frac{1}{2}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\varepsilon}H_\gamma{}^{\mu\zeta}\nabla_\zeta H_{\epsilon\varepsilon\mu}\Big].\end{aligned}$$

$$\begin{aligned}\mathbf{S}_{2e}^{(2)} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\Big[&-\frac{1}{12}H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}H_{\beta\theta}{}^{\lambda}H_\gamma{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau}\\ &+\frac{1}{4}H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}H_\gamma{}^{\kappa\lambda}H_\theta{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau} - 2H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta\theta}{}^{\lambda\mu}R_{\gamma\lambda\kappa\mu} + 2H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}R_{\alpha\beta\theta}{}^{\mu}R_{\gamma\mu\kappa\lambda}\\ &-2H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}R_\beta{}^{\lambda}{}_\gamma{}^{\mu}R_{\theta\lambda\kappa\mu} - \frac{1}{2}H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}\nabla_\alpha H_{\theta\beta}{}^{\mu}\nabla_\lambda H_{\gamma\kappa\mu} + \frac{1}{4}H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_\kappa H_{\theta\lambda\mu}\nabla^\mu H_{\beta\gamma}{}^{\lambda}\\ &+H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_\mu H_{\gamma\kappa\lambda}\nabla^\mu H_{\beta\theta}{}^{\lambda}\Big].\end{aligned}$$

Frame Trans.

To compare the couplings with string theory S-matrix elements → String Frame → $G^{s}_{\mu\nu} = e^{\gamma\phi} G^{E}_{\mu\nu}$, $\gamma = 4/(D-2)$ → Einstein Frame

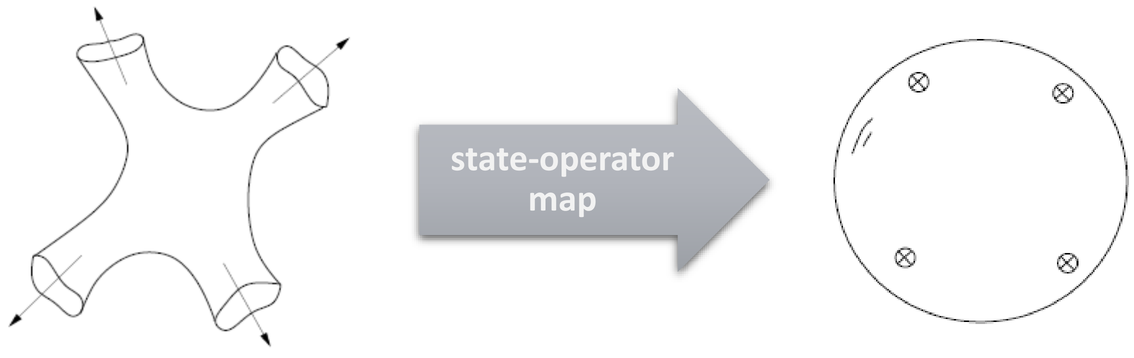
String frame

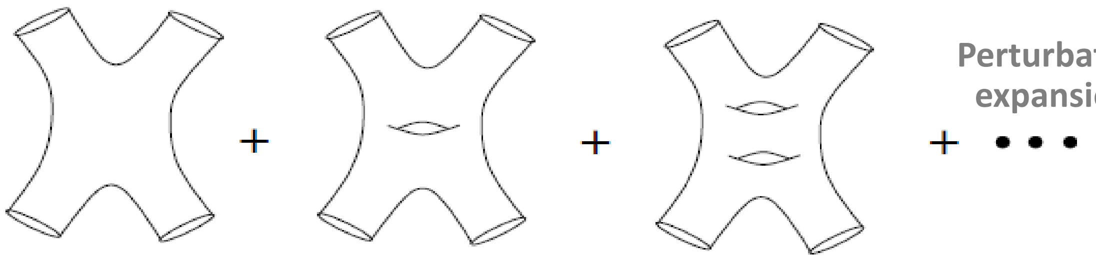
$$S^{(0)} = \frac{1}{2\kappa^2}\int d^D x e^{-2\phi}\sqrt{-G}\left[R + 4(\partial\phi)^2 - \frac{1}{12}H^2\right]$$

Einstein frame

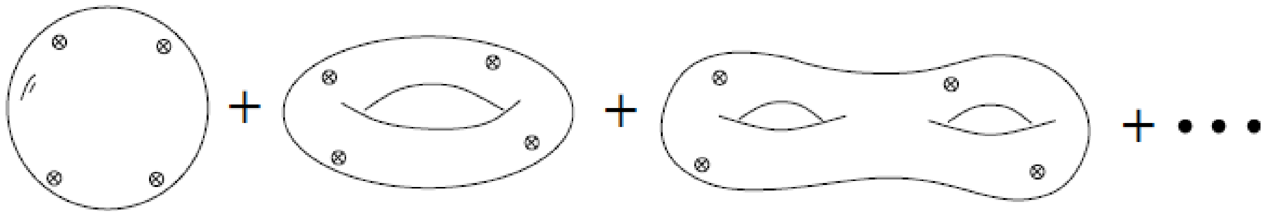
$$S^{(0)} = \frac{1}{2\kappa^2}\int d^D x\sqrt{-G}\left[R - \gamma(\partial\phi)^2 - \frac{1}{12}e^{-2\gamma\phi}H^2\right]$$

String theory scattering amplitude (close string)





Perturbative expansion



$$S_{\text{string}} = S_{\text{Poly}} + \lambda\chi$$

$$\chi = \frac{1}{4\pi}\int d^2\sigma\ \sqrt{g}R$$

Euler num.

Topological invariant

(Not depend on metric)

$$\chi = 2 - 2h = 2(1-g)$$

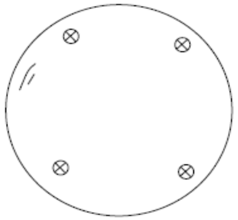
$$\sum_{\substack{\text{topologies}\\ \text{metrics}}} e^{-S_{\text{string}}} \sim \sum_{\text{topologies}} e^{-2\lambda(1-g)} \int \mathcal{D}X\mathcal{D}g\ e^{-S_{\text{Poly}}}$$

String theory scattering amplitude (close string)

S-matrix elements in 26D

$$\mathcal{A}^{(m)}(\Lambda_i, p_i) = \sum_{Topologies} g_s^{-\chi} \frac{1}{Vol} \int \mathcal{D}X\mathcal{D}g \, e^{-S_{Poly}} \prod_{i=1}^{m} V_{\Lambda_i}(p_i)$$

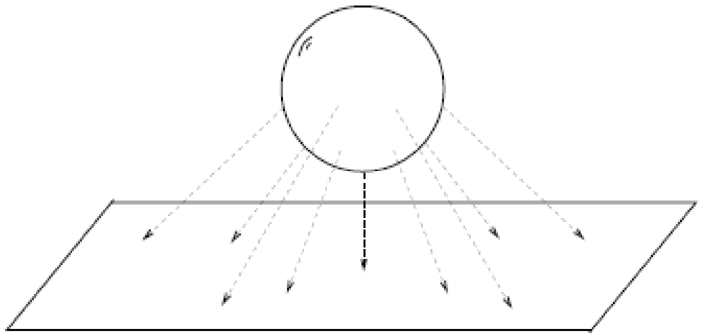
2D correlation function in CFT



$\chi = 2 - 2h = 2(1 - g)$

Sphere: $g = 0 \rightarrow \chi = 2 \qquad 1/g_s^2$

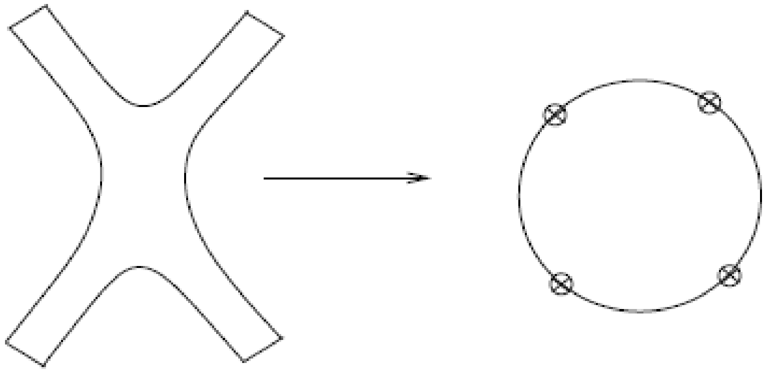
$$\mathcal{A}^{(m)} = \frac{1}{g_s^2} \frac{1}{Vol} \int \mathcal{D}X\mathcal{D}g \, e^{-S_{Poly}} \prod_{i=1}^{m} V_{\Lambda_i}(p_i)$$



Any metric on the sphere → Conformally → Flat metric on the plane

$$\mathcal{A}^{(m)} = \frac{1}{g_s^2} \frac{1}{Vol(SL(2;C))} \int \mathcal{D}X \, e^{-S_{Poly}} \prod_{i=1}^{m} V(p_i)$$

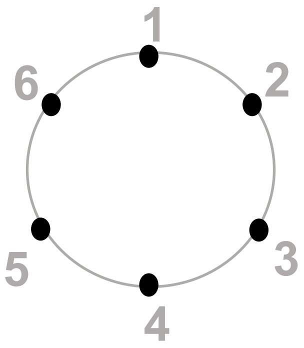
String theory scattering amplitude (open string)



$$A^{(m)}_{\text{closed}} = \left(\frac{1}{2}i\right)^{m-3} \pi\kappa^{m-2} \sum_{P,P'} A^{(m)}_{\text{open}}(P)\bar{A}^{(m)}_{\text{open}}(P')\mathrm{e}^{i\pi F(P,P')}$$

$$A^{(3)}_{\text{closed}} = \pi\kappa A^{(3)}_{\text{open}}\bar{A}^{(3)}_{\text{open}}$$

$$A^{(4)}_{\text{closed}} = -\pi\kappa^2 \sin\left(\pi\tfrac{1}{2}\alpha' k_2\cdot k_3\right) A^{(4)}_{\text{open}}\left(\alpha'\tfrac{1}{4}s,\ \alpha'\tfrac{1}{4}t\right)\bar{A}^{(4)}_{\text{open}}\left(\alpha'\tfrac{1}{4}t,\ \alpha'\tfrac{1}{4}u\right)$$



?

$$\begin{aligned}A^{(6)}_{\text{closed}} &= -\pi\kappa^4 A^{(6)}_{\text{open}}(123456)\sin(\pi k_1\cdot k_2)\sin(\pi k_4\cdot k_5)\\ &\times\left\{\bar{A}^{(6)}_{\text{open}}(215346)\sin(\pi k_3\cdot k_5) + \bar{A}^{(6)}_{\text{open}}(215436)\sin\left(\pi k_3(k_4+k_5)\right)\right\}\\ &+ \text{permutations of }(234)\end{aligned}$$

H. Kawai, D. C. Lewellen and S. H. H. Tye, Nucl. Phys. B **269**, 1 (1986)

String theory scattering amplitude (open string)

$$A^{(m)}_{open}(tachyon) \sim \int_{-\infty}^{\infty} dx_1 \dots dx_m \langle V(k_1,x_1)V(k_2,x_2)\dots V(k_m,x_m)\rangle \qquad V(k,x) = :e^{ik.X(x)}:$$

$$A^{(m)}_{open}(tachyon) \sim \int_{-\infty}^{\infty} dx_1 \dots dx_m \prod_{m\geq j>i\geq 1} |x_i - x_j|^{k_i.k_j}$$

$$A^{(m)}_{open}(tachyon) = \int_{-\infty}^{\infty} dx_1 \dots dx_m \frac{|(x_a - x_b)(x_b - x_c)(x_c - x_a)|}{dx_a dx_b dx_c} \prod_{m\geq j>i\geq 1} |x_i - x_j|^{k_i.k_j}$$

$$massless\ vector\ vertex \ \rightarrow\ V(\xi,k;x) = i\xi^\mu : \partial_x X^\mu e^{ik.X(x)} :$$

$$A^{(m)}_{\mu_1\dots\mu_m}\xi_1^{\mu_1}\dots\xi_m^{\mu_m} = \int_{-\infty}^{\infty} dx_1 \dots dx_m \frac{|(x_a - x_b)(x_b - x_c)(x_c - x_a)|}{dx_a\, dx_b\, dx_c} \times \prod_{m\geq j>i\geq 1} |x_i - x_j|^{k_i.k_j}\ exp\left[\sum_{j>i}\frac{\xi_i.\xi_j}{(x_i - x_j)^2} - \sum_{i\neq j}\frac{k_i.\xi_j}{(x_i - x_j)}\right]$$

6 open strings integration

$$\boldsymbol{x_1 = x_a = -\infty, \qquad x_2 = x_b = 0, \qquad x_3 = x_c = 1}$$

$$|(-\infty - \mathbf{0})(\mathbf{0} - \mathbf{1})(\mathbf{1} + \infty)|$$

$$\int_{-\infty}^{\infty} dx_1 \ldots dx_m \frac{|(x_a - x_b)(x_b - x_c)(x_c - x_a)|}{dx_a\, dx_b\, dx_c} \times \prod_{m \geq j > i \geq 1} |x_i - x_j|^{k_i . k_j} \; exp\left[\sum_{j>i} \frac{\xi_i . \xi_j}{(x_i - x_j)^2}\right]$$

$$|x_۱ - x_۲|^{k_۱.k_۲}\,|x_۱ - x_۳|^{k_۱.k_۳}\,|x_۱ - x_۴|^{k_۱.k_۴}\,|x_۱ - x_۵|^{k_۱.k_۵}\,|x_۱ - x_۶|^{k_۱.k_۶}$$
$$|x_۲ - x_۴|^{k_۲.k_۴}\,|x_۲ - x_۵|^{k_۲.k_۵}\,|x_۲ - x_۶|^{k_۲.k_۶}\,|x_۳ - x_۴|^{k_۳.k_۴}\,|x_۳ - x_۵|^{k_۳.k_۵}$$
$$|x_۳ - x_۶|^{k_۳.k_۶}|x_۴ - x_۵|^{k_۴.k_۵}\,|x_۴ - x_۶|^{k_۴.k_۶}\,|x_۵ - x_۶|^{k_۵.k_۶}$$

$$|x_۴|^{k_۲.k_۴}\,|x_۵|^{k_۲.k_۵}\,|x_۶|^{k_۲.k_۶}\,|۱ - x_۴|^{k_۳.k_۴}\,|۱ - x_۵|^{k_۳.k_۵}$$
$$|۱ - x_۶|^{k_۳.k_۶}|x_۴ - x_۵|^{k_۴.k_۵}\,|x_۴ - x_۶|^{k_۴.k_۶}\,|x_۵ - x_۶|^{k_۵.k_۶}$$

9 independent kinematic invariants

$$s_۱ = k_۲k_۴\,,\;\; s_۲ = k_۲k_۵\,,\;\; s_۳ = k_۲k_۶\,,\;\; s_۴ = k_۳k_۴\,,\;\; s_۵ = k_۳k_۵\,,\;\; s_۶ = k_۳k_۶$$
$$s_۷ = k_۴k_۵\,,\;\; s_۸ = k_۴k_۶\,,\;\; s_۹ = k_۵k_۶$$

6 open strings integration

$$|x_4|^{k_2.k_4}\,|x_5|^{k_2.k_5}\,|x_6|^{k_2.k_6}\,|1-x_4|^{k_3.k_4}\,|1-x_5|^{k_3.k_5}$$

$$|1-x_6|^{k_3.k_6}\,|x_4-x_5|^{k_4.k_5}\,|x_4-x_6|^{k_4.k_6}\,|x_5-x_6|^{k_5.k_6}$$

$$\int_{-\infty}^{\infty} dx_1 \ldots dx_m \frac{|(x_a-x_b)(x_b-x_c)(x_c-x_a)|}{dx_a\, dx_b\, dx_c} \times \prod_{m\geq j>i\geq 1} |x_i-x_j|^{k_i.k_j} \quad exp\left[\sum_{j>i} \frac{\xi_i.\xi_j}{(x_i-x_j)^2}\right]$$

$$\left(\frac{\xi_1\xi_2}{x_{12}^2}\frac{\xi_3\xi_4}{x_{34}^2}\frac{\xi_5\xi_6}{x_{56}^2}\right)+\left(\frac{\xi_1\xi_3}{x_{13}^2}\frac{\xi_2\xi_5}{x_{25}^2}\frac{\xi_4\xi_6}{x_{46}^2}\right)+\left(\frac{\xi_1\xi_2}{x_{12}^2}\frac{\xi_3\xi_6}{x_{36}^2}\frac{\xi_4\xi_5}{x_{45}^2}\right)$$
$$+\left(\frac{\xi_1\xi_3}{x_{13}^2}\frac{\xi_2\xi_4}{x_{24}^2}\frac{\xi_5\xi_6}{x_{56}^2}\right)+\left(\frac{\xi_1\xi_3}{x_{13}^2}\frac{\xi_2\xi_5}{x_{25}^2}\frac{\xi_4\xi_6}{x_{46}^2}\right)+\left(\frac{\xi_1\xi_3}{x_{13}^2}\frac{\xi_2\xi_6}{x_{26}^2}\frac{\xi_4\xi_5}{x_{45}^2}\right)$$
$$+\left(\frac{\xi_1\xi_4}{x_{14}^2}\frac{\xi_2\xi_3}{x_{23}^2}\frac{\xi_5\xi_6}{x_{56}^2}\right)+\left(\frac{\xi_1\xi_4}{x_{14}^2}\frac{\xi_2\xi_5}{x_{25}^2}\frac{\xi_3\xi_6}{x_{36}^2}\right)+\left(\frac{\xi_1\xi_4}{x_{14}^2}\frac{\xi_2\xi_6}{x_{26}^2}\frac{\xi_3\xi_5}{x_{35}^2}\right)$$
$$+\left(\frac{\xi_1\xi_5}{x_{15}^2}\frac{\xi_2\xi_3}{x_{23}^2}\frac{\xi_4\xi_6}{x_{46}^2}\right)+\left(\frac{\xi_1\xi_5}{x_{15}^2}\frac{\xi_2\xi_4}{x_{24}^2}\frac{\xi_3\xi_6}{x_{36}^2}\right)+\left(\frac{\xi_1\xi_5}{x_{15}^2}\frac{\xi_2\xi_6}{x_{26}^2}\frac{\xi_3\xi_4}{x_{34}^2}\right)$$
$$+\left(\frac{\xi_1\xi_6}{x_{16}^2}\frac{\xi_2\xi_3}{x_{23}^2}\frac{\xi_4\xi_5}{x_{45}^2}\right)+\left(\frac{\xi_1\xi_6}{x_{16}^2}\frac{\xi_2\xi_4}{x_{24}^2}\frac{\xi_3\xi_5}{x_{35}^2}\right)+\left(\frac{\xi_1\xi_6}{x_{16}^2}\frac{\xi_2\xi_5}{x_{25}^2}\frac{\xi_3\xi_4}{x_{34}^2}\right)$$

$$x_{ij}^2 = (x_i - x_j)^2$$

$$\left(\frac{\xi_1\xi_2}{1}\frac{\xi_3\xi_4}{(1-x_4)^2}\frac{\xi_5\xi_6}{(x_5-x_6)^2}\right)+\left(\frac{\xi_1\xi_3}{1}\frac{\xi_2\xi_5}{x_5^2}\frac{\xi_4\xi_6}{(x_4-x_6)^2}\right)+\left(\frac{\xi_1\xi_2}{1}\frac{\xi_3\xi_6}{(1-x_6)^2}\frac{\xi_4\xi_5}{(x_4-x_5)^2}\right)$$
$$+\left(\frac{\xi_1\xi_3}{1}\frac{\xi_2\xi_4}{x_4^2}\frac{\xi_5\xi_6}{(x_5-x_6)^2}\right)+\left(\frac{\xi_1\xi_3}{1}\frac{\xi_2\xi_5}{x_5^2}\frac{\xi_4\xi_6}{(x_4-x_6)^2}\right)+\left(\frac{\xi_1\xi_3}{1}\frac{\xi_2\xi_6}{x_6^2}\frac{\xi_4\xi_5}{(x_4-x_5)^2}\right)$$
$$+\left(\frac{\xi_1\xi_4}{1}\frac{\xi_2\xi_3}{1}\frac{\xi_5\xi_6}{(x_5-x_6)^2}\right)+\left(\frac{\xi_1\xi_4}{1}\frac{\xi_2\xi_5}{x_5^2}\frac{\xi_3\xi_6}{(1-x_6)^2}\right)+\left(\frac{\xi_1\xi_4}{1}\frac{\xi_2\xi_6}{x_6^2}\frac{\xi_3\xi_5}{(1-x_5)^2}\right)$$
$$+\left(\frac{\xi_1\xi_5}{1}\frac{\xi_2\xi_3}{1}\frac{\xi_4\xi_6}{(x_4-x_6)^2}\right)+\left(\frac{\xi_1\xi_5}{1}\frac{\xi_2\xi_4}{x_4^2}\frac{\xi_3\xi_6}{(1-x_6)^2}\right)+\left(\frac{\xi_1\xi_5}{1}\frac{\xi_2\xi_6}{x_6^2}\frac{\xi_3\xi_4}{(1-x_4)^2}\right)$$
$$+\left(\frac{\xi_1\xi_6}{1}\frac{\xi_2\xi_3}{1}\frac{\xi_4\xi_5}{(x_4-x_5)^2}\right)+\left(\frac{\xi_1\xi_6}{1}\frac{\xi_2\xi_4}{x_4^2}\frac{\xi_3\xi_5}{(1-x_5)^2}\right)+\left(\frac{\xi_1\xi_6}{1}\frac{\xi_2\xi_5}{x_5^2}\frac{\xi_3\xi_4}{(1-x_4)^2}\right)$$

6 open strings integration

$$\int_{-\infty}^{\infty} dx_1 \ldots dx_m \frac{|(x_a - x_b)(x_b - x_c)(x_c - x_a)|}{dx_a\, dx_b\, dx_c} \times \prod_{m \geq j > i \geq 1} |x_i - x_j|^{k_i . k_j} \; exp\left[\sum_{j>i} \frac{\xi_i . \xi_j}{(x_i - x_j)^2}\right]$$

$$\Xi_1 := (\xi_1\xi_2)\,(\xi_3\xi_4)\,(\xi_5\xi_6)\;,\quad \Xi_2 := (\xi_1\xi_2)\,(\xi_3\xi_5)\,(\xi_4\xi_6)\;,\quad \Xi_3 := (\xi_1\xi_2)\,(\xi_3\xi_6)\,(\xi_4\xi_5),$$
$$\Xi_4 := (\xi_1\xi_3)\,(\xi_2\xi_4)\,(\xi_5\xi_6)\;,\quad \Xi_5 := (\xi_1\xi_3)\,(\xi_2\xi_5)\,(\xi_4\xi_6)\;,\quad \Xi_6 := (\xi_1\xi_3)\,(\xi_2\xi_6)\,(\xi_4\xi_5),$$
$$\Xi_7 := (\xi_1\xi_4)\,(\xi_2\xi_3)\,(\xi_5\xi_6)\;,\quad \Xi_8 := (\xi_1\xi_4)\,(\xi_2\xi_5)\,(\xi_3\xi_6)\;,\quad \Xi_9 := (\xi_1\xi_4)\,(\xi_2\xi_6)\,(\xi_3\xi_5),$$
$$\Xi_{10} := (\xi_1\xi_5)\,(\xi_2\xi_3)\,(\xi_4\xi_6)\;,\quad \Xi_{11} := (\xi_1\xi_5)\,(\xi_2\xi_4)\,(\xi_3\xi_6)\;,\quad \Xi_{12} := (\xi_1\xi_5)\,(\xi_2\xi_6)\,(\xi_3\xi_4),$$
$$\Xi_{13} := (\xi_1\xi_6)\,(\xi_2\xi_3)\,(\xi_4\xi_5)\;,\quad \Xi_{14} := (\xi_1\xi_6)\,(\xi_2\xi_4)\,(\xi_3\xi_5)\;,\quad \Xi_{15} := (\xi_1\xi_6)\,(\xi_2\xi_5)\,(\xi_3\xi_4).$$

$$\int dx_4\, dx_5\, dx_6\; |x_4|^{\alpha_{24}} |x_5|^{\alpha_{25}} |x_6|^{\alpha_{26}} |1 - x_4|^{\alpha_{34}} |1 - x_5|^{\alpha_{35}} |1 - x_6|^{\alpha_{36}} |x_4 - x_5|^{\alpha_{45}} |x_4 - x_6|^{\alpha_{46}} |x_5 - x_6|^{\alpha_{56}}$$

$$\alpha_{24} = s_1 + n_{24}\,, \quad \alpha_{25} = s_2 + n_{25}\,, \quad \alpha_{26} = s_3 + n_{26}$$
$$\alpha_{34} = s_4 + n_{34}\,, \quad \alpha_{35} = s_5 + n_{35}\,, \quad \alpha_{36} = s_6 + n_{36}$$
$$\alpha_{45} = s_7 + n_{45}\,, \quad \alpha_{46} = s_8 + n_{46}\,, \quad \alpha_{56} = s_9 + n_{56}$$

4,5 & 6 open strings

$$B(a,b) = \int_0^1 dx\ x^a\ (1-x)^b = \frac{1}{1+a}\ {}_2F_1\left[1+a, -b, 2+a; 1\right]$$
$$= \frac{\Gamma(1+a)\ \Gamma(1+b)}{\Gamma(2+a+b)}\ ,\quad \mathrm{Re}\ a > -1\ ,\ \mathrm{Re}\ b > -1$$

$$C(a,b,c,d,e) := \int_0^1 dx \int_0^1 dy\ x^a\ y^b\ (1-x)^c\ (1-y)^d\ (1-xy)^e$$

$$C(a,b,c,d,e) = \frac{\Gamma(1+a)\ \Gamma(1+b)\ \Gamma(1+c)\ \Gamma(1+d)}{\Gamma(2+a+c)\ \Gamma(2+b+d)}\ {}_3F_2\left[\begin{matrix}1+a,\ 1+b,\ -e \\ 2+a+c,\ 2+b+d\end{matrix}\ ;\ 1\right]$$

with $\mathrm{Re}(a), \mathrm{Re}(b), \mathrm{Re}(c), \mathrm{Re}(d) > -1$

$$x_{۴} = x, \qquad x_{۵} = xy, \qquad x_{۶} = xyz$$
$$x \to \frac{۱}{x}\ ,\ y \to \frac{۱}{y}\ ,\ z \to \frac{۱}{z}$$

$$F\left[\begin{matrix}a,b,d,e,g \\ c,f,h,j\end{matrix}\right] := \int_0^1 dx \int_0^1 dy \int_0^1 dz$$
$$\times\ x^a\ y^b\ z^c\ (1-x)^d\ (1-y)^e\ (1-z)^f\ (1-xy)^g\ (1-yz)^h\ (1-xyz)^j$$

$$F\left[\begin{matrix}a,b,d,e,g \\ c,f,h,j\end{matrix}\right] = \frac{\Gamma(1+a)\ \Gamma(1+b)\ \Gamma(1+c)\ \Gamma(1+d)\ \Gamma(1+e)\ \Gamma(1+f)}{\Gamma(2+a+d)\ \Gamma(2+b+e)\ \Gamma(2+c+f)}$$
$$\times\ F^{(3)}\left[\begin{matrix}1+b :: 1; 1+c; 1+a : -g, 1; -h, 1; -j, 1 \\ 2+b+e :: 1; 2+c+f; 2+a+d : 1; 1; 1\end{matrix}\ ;\ 1,1,1\right]$$

Triple Hypergeometric Function

$$a = -4 - \alpha_{24} - \alpha_{25} - \alpha_{26} - \alpha_{34} - \alpha_{35} - \alpha_{36} - \alpha_{45} - \alpha_{46} - \alpha_{56}\ ,$$
$$b = -3 - \alpha_{25} - \alpha_{26} - \alpha_{35} - \alpha_{36} - \alpha_{45} - \alpha_{46} - \alpha_{56}\ ,$$
$$c = -2 - \alpha_{26} - \alpha_{36} - \alpha_{46} - \alpha_{56}\ ,$$
$$d = \alpha_{34}\quad ,\quad e = \alpha_{45}\quad ,\quad f = \alpha_{56}$$
$$g = \alpha_{35}\quad ,\quad h = \alpha_{46}\quad ,\quad j = \alpha_{36}\ ,$$

Triple hypergeometric function integration (*symmetries*)

۱) $F[a, b-1, c, d-2, e, f-2, g, h, j]\ \Xi_1$

۲) $F[a, b+1, c, d, e, f, g-2, h-2, j]\ \Xi_2$

۳) $F[a, b+1, c, d, e-2, f, g, h, j-2]\ \Xi_3$

۴) $F[a, b-1, c, d, e, f-2, g, h, j]\ \Xi_4$

۵) $F[a, b+1, c, d, e, f, g, h-2, j]\ \Xi_5$

۶) $F[a, b+1, c, d, e-2, f, g, h, j]\ \Xi_6$

۷) $F[a-2, b-1, c, d, e, f-2, g, h, j]\ \Xi_7$

۸) $F[a, b+1, c, d, e, f, g, h, j-2]\ \Xi_8$

۹) $F[a, b+1, c, d, e, f, g-2, h, j]\ \Xi_9$

۱۰) $F[a-2, b-1, c, d, e, f, g, h-2, j]\ \Xi_{10}$

۱۱) $F[a-1, b, c, d, e, f, g, h, j-2]\ \Xi_{11}$

۱۲) $F[a, b-1, c, d-2, e, f, g, h, j]\ \Xi_{12}$

۱۳) $F[a-2, b-1, c-2, d, e-2, f, g, h, j]\ \Xi_{13}$

۱۴) $F[a, b-1, c-2, d, e, f, g-2, h, j]\ \Xi_{14}$

۱۵) $F[a, b-1, c-2, d-2, e, f, g, h, j]\ \Xi_{15}$

واگرایی انتگرالده‌ها

$$\int_0^1 dx \int_0^1 dy \int_0^1 dz\, x^{a-1} y^b z^c (1-x)^d (1-y)^e (1-z)^f (1-xy)^g (1-yz)^h (1-xyz)^j$$

الف) تقارن در ظاهر انتگرالده‌ها

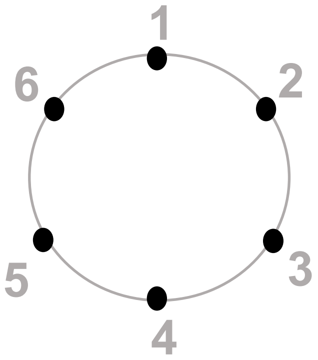
$a \leftrightarrow c$ $\quad d \leftrightarrow f$ $\quad g \leftrightarrow h$

$$W = x^a y^b z^c (1-x)^d (1-y)^e (1-z)^f (1-xy)^g (1-yz)^h (1-xyz)^j$$

$$\underbrace{\iiint_0^1 dx\, dy\, dz \frac{W}{y(1-z)^2}}_{\text{4 انتگرال}} \equiv \underbrace{\iiint_0^1 dx\, dy\, dz \frac{W}{y(1-x)^2}}_{\text{12 انتگرال}}$$

10 ⇄ 14 $\quad$ 7 ⇄ 15 $\quad$ 4 ⇄ 12

Triple hypergeometric function integration (*symmetries*)



ب) تقارن چرخشی(Cyclic)

۱) $F[a,b-1,c,d-2,e,f-2,g,h,j]\ \Xi_1$

۲) $F[a,b+1,c,d,e,f,g-2,h-2,j]\ \Xi_2$

۳) $F[a,b+1,c,d,e-2,f,g,h,j-2]\ \Xi_3$

۴) $F[a,b-1,c,d,e,f-2,g,h,j]\ \Xi_4$

۵) $F[a,b+1,c,d,e,f,g,h-2,j]\ \Xi_5$

۶) $F[a,b+1,c,d,e-2,f,g,h,j]\ \Xi_6$

۷) $F[a-2,b-1,c,d,e,f-2,g,h,j]\ \Xi_7$

۸) $F[a,b+1,c,d,e,f,g,h,j-2]\ \Xi_8$

۹) $F[a,b+1,c,d,e,f,g-2,h,j]\ \Xi_9$

۱۰) $F[a-2,b-1,c,d,e,f,g,h-2,j]\ \Xi_{10}$

۱۱) $F[a-1,b,c,d,e,f,g,h,j-2]\ \Xi_{11}$

۱۲) $F[a,b-1,c,d-2,e,f,g,h,j]\ \Xi_{12}$

۱۳) $F[a-2,b-1,c-2,d,e-2,f,g,h,j]\ \Xi_{13}$

۱۴) $F[a,b-1,c-2,d,e,f,g-2,h,j]\ \Xi_{14}$

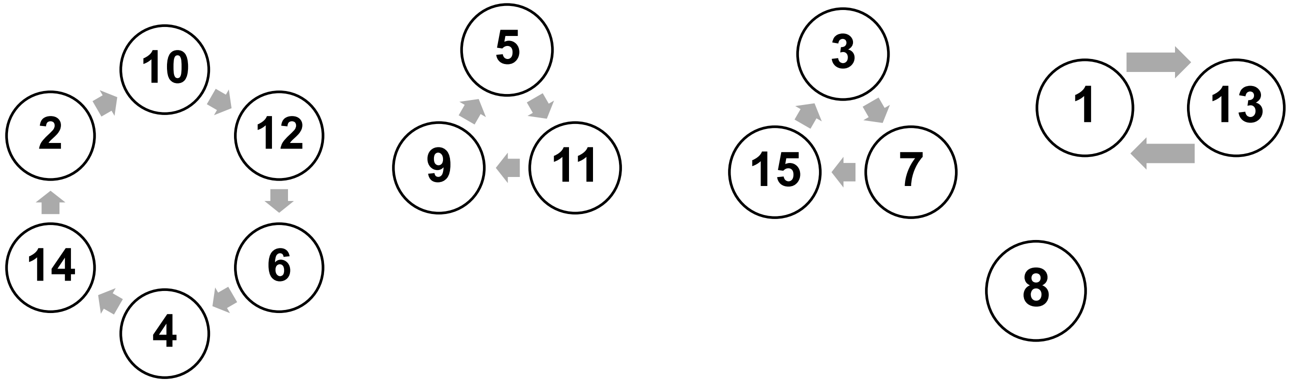
۱۵) $F[a,b-1,c-2,d-2,e,f,g,h,j]\ \Xi_{15}$

$$\Xi_5 := (\xi_1\xi_3)\ (\xi_2\xi_5)\ (\xi_4\xi_6)$$

$$1 \to 2,\ 2 \to 3,\ 3 \to 4,\ 4 \to 5,\ 5 \to 6,\ 6 \to 1 \quad ,$$

$$\begin{cases} s_1 \to s_5 \\ s_2 \to s_6 \\ s_3 \to s_1 + s_2 + s_3 + s_7 + s_8 + s_9 \\ s_4 \to s_7 \\ s_5 \to s_8 \\ s_6 \to -s_1 - s_4 - s_7 - s_8 \\ s_7 \to s_9 \\ s_8 \to -s_2 - s_5 - s_7 - s_9 \\ s_9 \to -s_3 - s_6 - s_8 - s_9\ , \end{cases}$$

$$\Xi_{11} := (\xi_1\xi_5)\ (\xi_2\xi_4)\ (\xi_3\xi_6)$$



Triple hypergeometric function integration

$$\int_0^1 dx \int_0^1 dy \int_0^1 dz\, x^{a-1}\, y^b\, z^c\, (1-x)^d\, (1-y)^e\, (1-z)^f\, (1-xy)^g\, (1-yz)^h\, (1-xyz)^j$$

$x \to 0$

$$\underbrace{\int_0^1 dx \int_0^1 dy \int_0^1 dz\; x^{a-1}\, y^b\, z^c\, (1-x)^d\, (1-y)^e\, (1-z)^f\, (1-yz)^h \left[(1-xy)^g (1-xyz)^j - 1\right]}_{\text{I}}$$

$$+ \underbrace{\int_0^1 dx \int_0^1 dy \int_0^1 dz\; x^{a-1}\, y^b\, z^c\, (1-x)^d\, (1-y)^e\, (1-z)^f\, (1-yz)^h}_{\text{II}}$$

$$\underbrace{\int_0^1 dx\; x^{a-1}\, (1-x)^d}_{\text{انتگرال بتا}} \times \underbrace{\int_0^1 dy \int_0^1 dz\;\; y^b\, z^c\;\, (1-y)^e\, (1-z)^f\, (1-yz)^h}_{\text{انتگرال هایپرژئومتریک}}$$

Triple hypergeometric function integration

$$x^a\, y^{b+١}\, z^c\, (١-x)^d\, (١-y)^e\, (١-z)^f\, (١-xy)^g\, (١-yz)^{h-٢} (١-xyz)^j =$$

$$yz \to ١ \overset{\text{یا}}{\Rightarrow} y \to \frac{١}{z} \xRightarrow{0\le y,z\le ١} y \to z \to ١$$

در حد $\mathbf{yz \to 1}$ به سمت صفر میل می‌کند

$$\underbrace{x^a\, y^{b+١}\, (١-x)^d\, (١-y)^e\, (١-xy)^g\, (١-yz)^{h-٢}\, \overbrace{[z^c\, (١-z)^f\, (١-xyz)^j - y^c\, (١-y)^f\, (١-x)^j]}}_{\text{I}} +$$

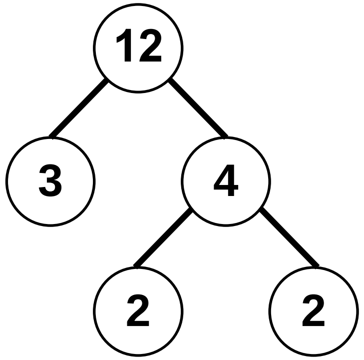
$$\underbrace{x^a\, y^{b+c+١}\, (١-x)^{d+j}\, (١-y)^{e+f}\, (١-xy)^g\, (١-yz)^{h-٢}}_{\text{II}}$$

روش حل انتگرال ساده شده II

۱) انتگرال‌گیری مستقیم از متغییری که کمتری اثر را داشته باشد. (اگر از لحاظ محاسبه‌ی کامپیوتری این امکان فراهم باشد)

۲) ساده‌سازی به روش جز به جز

۳) ساده‌سازی دوباره به روشی که در ابتدا و روی انتگرالده اولیه انجام دادیم.



Triple hypergeometric function integration

$$x^{a} y^{b-۱} z^{c} (۱-x)^{d} (۱-y)^{e} (۱-z)^{f-۲} (۱-xy)^{g} (۱-yz)^{h} (۱-xyz)^{j} =$$

$$\underbrace{x^{a} y^{b-۱} (۱-x)^{d} (۱-y)^{e} (۱-z)^{f-۲} (۱-xy)^{g} \overbrace{[z^{c}-۱]}^{\substack{\text{عبارت} \\ \mathbf{z \to 1 \Longrightarrow 0}}} \overbrace{[(۱-yz)^{h}(۱-xyz)^{j}-(۱-y)^{h}(۱-xy)^{j}]}^{\text{در حد } \mathbf{y \to 0} \text{ یا } \mathbf{z \to 1} \text{ به سمت صفر میل می‌کند}}}_{\mathbf{I}} -$$

$$\underbrace{x^{a} y^{b-۱} (۱-x)^{d} (۱-y)^{e+h} (۱-z)^{f-۲} (۱-xy)^{g+j}}_{\mathbf{II}}$$

$$+ \underbrace{x^{a} y^{b-۱} z^{c} (۱-x)^{d} (۱-y)^{e+h} (۱-z)^{f-۲} (۱-xy)^{g+j}}_{\mathbf{III}}$$

$$+ \underbrace{x^{a} y^{b-۱} (۱-x)^{d} (۱-y)^{e} (۱-z)^{f-۲} (۱-xy)^{g} (۱-yz)^{h} (۱-xyz)^{j}}_{\mathbf{IV}}$$

Triple hypergeometric function integration

$$x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^e\, (۱-z)^{f-۲}\, (۱-xy)^g\, (۱-yz)^h\, (۱-xyz)^j =$$

$$\underbrace{x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^e\, (۱-z)^{f-۲}\, (۱-xy)^g\, [(۱-yz)^h-(۱-y)^h]\,[(۱-xyz)^j-(۱-xy)^j]}_{①}$$

$$\underbrace{-\,x^a\, y^{b-۱}\, (۱-z)^{f-۲}\, (۱-xy)^{g+j}\, (۱-x)^d\, (۱-y)^{e+h}}_{②}$$

$$\underbrace{+\,x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^e\, (۱-z)^{f-۲}\, (۱-yz)^h\, (۱-xy)^{g+j}}_{③}$$

$$\underbrace{+\,x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^{e+h}\, (۱-z)^{f-۲}\, (۱-xy)^g\, (۱-xyz)^j}_{④}$$

انتگرال‌گیری جز به جز

$$\iint_0^۱ dx\, dy \ldots \int_0^۱ [dz\, (۱-z)^{f-۲}]\, (۱-yz)^h =$$

$$\left(\frac{(۱-z)^f (۱-yz)^h}{(۱-z)(۱-f)}\Bigg|_0^۱\right) \iint_0^۱ dx\, dy\;\; x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^e\, (۱-xy)^{g+j}$$

$$+\frac{h}{۱-f}\iiint_0^۱ dx\, dy\, dz\; x^a\, y^b\, (۱-x)^d\, (۱-y)^e\, (۱-z)^{f-۱}\, (۱-yz)^{h-۱}\, (۱-xy)^{g+j}$$

۲ مرتبه جز به جز

$$\frac{j\,(۱-j)}{f\,(۱-f)}\iiint_0^۱ dx\, dy\, dz\;\; x^{a+۲}\, y^{b+۱}\, (۱-x)^d\, (۱-y)^{e+h}\, (۱-z)^f\, (۱-xy)^g\, (۱-xyz)^{j-۲}$$

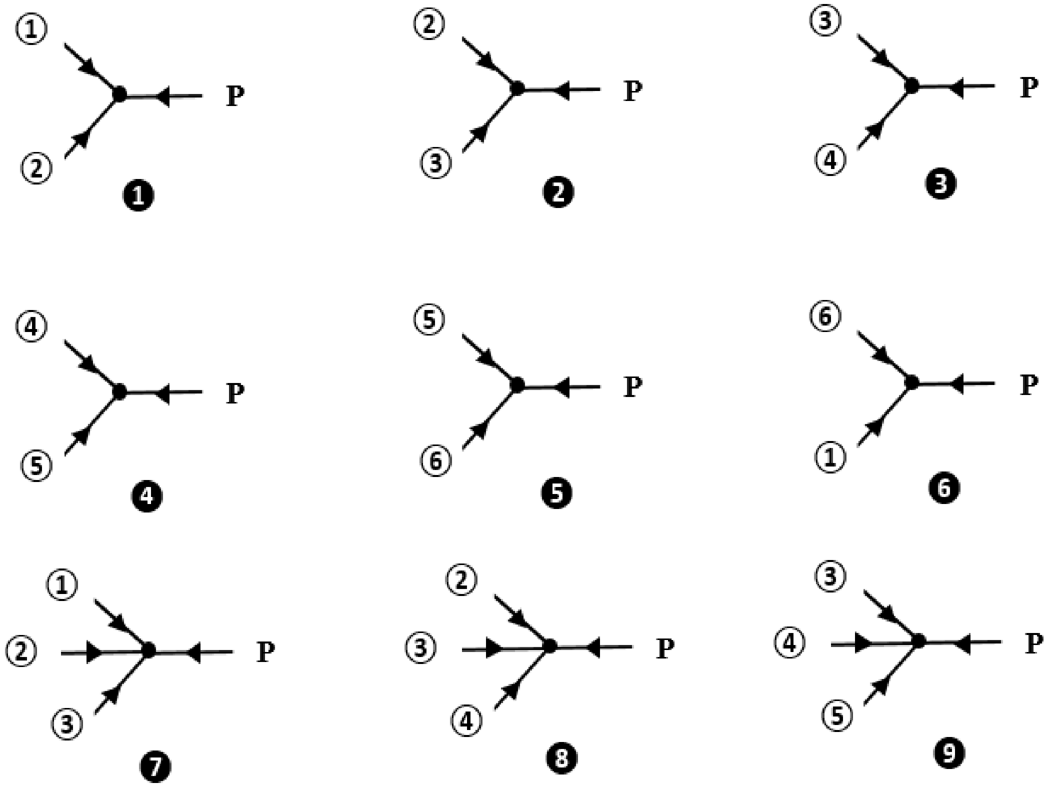
$$\left(\left(\frac{(۱-z)^f\, (۱-xyz)^j}{(۱-z)\,(۱-f)}\right)-\left(\frac{j\, x\, y\, (۱-z)^f\, (۱-xyz)^j}{f\,(۱-x\,y\,z)\,(۱-f)}\right)\right)\Bigg|_0^۱$$

$$\times\iint_0^۱ dx\, dy\, x^a\, y^{b-۱}\, (۱-x)^d\, (۱-y)^{e+h}\, (۱-xy)^g$$

Triple hypergeometric function integration

$$x^a\, y^b\, (۱-x)^d\, (۱-y)^e\, (۱-z)^{f-۱} (۱-yz)^{h-۱}\, (۱-xy)^{g+j} =$$

قطب غیر فیزیکی
و جواب ناهمخوان



نمودار ۱ $\rightarrow \quad k_p + k_۱ + k_۲ = 0 \quad \rightarrow \quad k_p^۲ = (k_۱ + k_۲)^۲$

$$k_p^۲ = ۲\, k_۱ \,.\, k_۲$$

$$k_p^۲ \propto (d+e+f+g+h+j)$$

❶ $\Rightarrow d+e+f+g+h+j\,,$ ❷ $\Rightarrow a\,,$ ❸ $\Rightarrow d\,,$ ❹ $\Rightarrow e\,,$ ❺ $\Rightarrow f$

❻ $\Rightarrow c\,,$ ❼ $\Rightarrow e+f+h\,,$ ❽ $\Rightarrow b\,,$ ❾ $\Rightarrow d+e+g$

6 open strings amplitude

$$\int_0^{١} dx \int_0^{١} dy \int_0^{١} dz\, x^a\, y^{b-١}\, z^c\, (١-x)^d\, (١-y)^e\, (١-z)^{f-٢}\, (١-xy)^g\, (١-yz)^h\, (١-xyz)^j$$

$$\begin{aligned} A4 = {} & \frac{a}{b} - \frac{c}{b} + \frac{d}{b} + \frac{a\,c}{b\,f} + \frac{c\,d}{b\,f} - \frac{f}{b} + \frac{h}{e+f+h} - \frac{a\,h}{f\,(e+f+h)} - \frac{d\,h}{f\,(e+f+h)} - \frac{g\,h}{f\,(e+f+h)} + \frac{j}{f} - \frac{h\,j}{f\,(e+f+h)} - \\ & \frac{1}{b\,\alpha} - \frac{c}{b\,f\,\alpha} + \frac{h}{f\,(e+f+h)\,\alpha} - \frac{a^2\,\alpha}{b} + \frac{a\,c\,\alpha}{b} - \frac{2\,a\,d\,\alpha}{b} + \frac{c\,d\,\alpha}{b} - \frac{d^2\,\alpha}{b} - \frac{a^2\,c\,\alpha}{b\,f} - \frac{2\,a\,c\,d\,\alpha}{b\,f} - \frac{c\,d^2\,\alpha}{b\,f} + \\ & \frac{a\,f\,\alpha}{b} - \frac{c\,f\,\alpha}{b} + \frac{d\,f\,\alpha}{b} - \frac{f^2\,\alpha}{b} - g\,\alpha - \frac{c\,g\,\alpha}{f} - \frac{a\,h\,\alpha}{e+f+h} - \frac{d\,h\,\alpha}{e+f+h} + \frac{a^2\,h\,\alpha}{f\,(e+f+h)} + \frac{2\,a\,d\,h\,\alpha}{f\,(e+f+h)} + \\ & \frac{d^2\,h\,\alpha}{f\,(e+f+h)} + \frac{f\,h\,\alpha}{e+f+h} - \frac{g\,h\,\alpha}{e+f+h} + \frac{2\,a\,g\,h\,\alpha}{f\,(e+f+h)} + \frac{2\,d\,g\,h\,\alpha}{f\,(e+f+h)} + \frac{g^2\,h\,\alpha}{f\,(e+f+h)} - \frac{2\,a\,j\,\alpha}{f} + \frac{b\,j\,\alpha}{f} - \\ & \frac{c\,j\,\alpha}{f} - \frac{2\,d\,j\,\alpha}{f} - \frac{g\,j\,\alpha}{f} - \frac{h\,j\,\alpha}{e+f+h} + \frac{2\,a\,h\,j\,\alpha}{f\,(e+f+h)} + \frac{2\,d\,h\,j\,\alpha}{f\,(e+f+h)} + \frac{2\,g\,h\,j\,\alpha}{f\,(e+f+h)} - \frac{j^2\,\alpha}{f} + \frac{h\,j^2\,\alpha}{f\,(e+f+h)} \end{aligned}$$

$R[1,2,3,4,5,6]=$

$$\begin{aligned} & B_1\Xi_1 + B_2\Xi_3 + B_3\Xi_3 + B_4\Xi_4 + B_5\Xi_5 \\ & + B_6\Xi_6 + B\Xi_7 + B_8\Xi_8 + B_9\Xi_9 \\ & + B_{10}\Xi_{10} + B_{11}\Xi_{11} + B_{12}\Xi_{12} \\ & + B_{13}\Xi_{13} + B_{14}\Xi_{14} + B_{15}\Xi_{15} \end{aligned}$$

$\mathbb{b}^1\mathbb{b}^2\mathbb{b}^3\mathbb{b}^4\mathbb{b}^5\mathbb{b}^6$ $\qquad$ $\boldsymbol{g, h, j = 0}$

$$\frac{2\,(a+d)\,(c+f)\;\mathbb{b}^{1AB}\;\mathbb{b}^{2}{}_{A}{}^{F}\;\mathbb{b}^{3}{}_{F}{}^{H}\;\mathbb{b}^{4}{}_{H}{}^{J}\;\mathbb{b}^{5}{}_{J}{}^{L}\;\mathbb{b}^{6}{}_{BL}}{b}$$

$$\frac{2\,(a+d)\,(c+f)\,(a-b+c+d+f)\;\mathbb{b}^{1AB}\;\mathbb{b}^{2}{}_{A}{}^{F}\;\mathbb{b}^{3}{}_{F}{}^{H}\;\mathbb{b}^{4}{}_{H}{}^{J}\;\mathbb{b}^{5}{}_{J}{}^{L}\;\mathbb{b}^{6}{}_{BL}}{b}$$

$$\frac{2\,(a+d)\,(c+f)\,(a^2+c^2+c\,d+d^2+c\,f+d\,f+f^2+a\,(-b+c+d+f)-b\,(c+d+e+f))\;\mathbb{b}^{1AB}\;\mathbb{b}^{2}{}_{A}{}^{F}\;\mathbb{b}^{3}{}_{F}{}^{H}\;\mathbb{b}^{4}{}_{H}{}^{J}\;\mathbb{b}^{5}{}_{J}{}^{L}\;\mathbb{b}^{6}{}_{BL}}{b}$$

$$\begin{aligned} A^{(6)}_{\text{closed}} = {} & -\pi\kappa^4 A^{(6)}_{\text{open}}(123456)\sin(\pi k_1\cdot k_2)\sin(\pi k_4\cdot k_5) \\ & \times\left\{\tilde{A}^{(6)}_{\text{open}}(215346)\sin(\pi k_3\cdot k_5) + \tilde{A}^{(6)}_{\text{open}}(215436)\sin\big(\pi k_3(k_4+k_5)\big)\right\} \\ & + \text{permutations of } (234) \end{aligned}$$

6 Abelian fields

R[1, 2, 3, 4, 5, 6] → open string → 2 مشتق

120

```
R[1, 2, 3, 4, 5, 6] + R[1, 2, 3, 4, 6, 5] + R[1, 2, 3, 5, 4, 6] + R[1, 2, 3, 5, 6, 4] + R[1, 2, 3, 6, 4, 5] +
 R[1, 2, 3, 6, 5, 4] + R[1, 2, 4, 3, 5, 6] + R[1, 2, 4, 3, 6, 5] + R[1, 2, 4, 5, 3, 6] + R[1, 2, 4, 5, 6, 3] +
 R[1, 2, 4, 6, 3, 5] + R[1, 2, 4, 6, 5, 3] + R[1, 2, 5, 3, 4, 6] + R[1, 2, 5, 3, 6, 4] + R[1, 2, 5, 4, 3, 6] +
 R[1, 2, 5, 4, 6, 3] + R[1, 2, 5, 6, 3, 4] + R[1, 2, 5, 6, 4, 3] + R[1, 2, 6, 3, 4, 5] + R[1, 2, 6, 3, 5, 4] +
 R[1, 2, 6, 4, 3, 5] + R[1, 2, 6, 4, 5, 3] + R[1, 2, 6, 5, 3, 4] + R[1, 2, 6, 5, 4, 3] + R[1, 3, 2, 4, 5, 6] +
 R[1, 3, 2, 4, 6, 5] + R[1, 3, 2, 5, 4, 6] + R[1, 3, 2, 5, 6, 4] + R[1, 3, 2, 6, 4, 5] + R[1, 3, 2, 6, 5, 4] +
 R[1, 3, 4, 2, 5, 6] + R[1, 3, 4, 2, 6, 5] + R[1, 3, 4, 5, 2, 6] + R[1, 3, 4, 5, 6, 2] + R[1, 3, 4, 6, 2, 5] +
 R[1, 3, 4, 6, 5, 2] + R[1, 3, 5, 2, 4, 6] + R[1, 3, 5, 2, 6, 4] + R[1, 3, 5, 4, 2, 6] + R[1, 3, 5, 4, 6, 2] +
 R[1, 3, 5, 6, 2, 4] + R[1, 3, 5, 6, 4, 2] + R[1, 3, 6, 2, 4, 5] + R[1, 3, 6, 2, 5, 4] + R[1, 3, 6, 4, 2, 5] +
 R[1, 3, 6, 4, 5, 2] + R[1, 3, 6, 5, 2, 4] + R[1, 3, 6, 5, 4, 2] + R[1, 4, 2, 3, 5, 6] + R[1, 4, 2, 3, 6, 5] +
 R[1, 4, 2, 5, 3, 6] + R[1, 4, 2, 5, 6, 3] + R[1, 4, 2, 6, 3, 5] + R[1, 4, 2, 6, 5, 3] + R[1, 4, 3, 2, 5, 6] +
 R[1, 4, 3, 2, 6, 5] + R[1, 4, 3, 5, 2, 6] + R[1, 4, 3, 5, 6, 2] + R[1, 4, 3, 6, 2, 5] + R[1, 4, 3, 6, 5, 2] +
 R[1, 4, 5, 2, 3, 6] + R[1, 4, 5, 2, 6, 3] + R[1, 4, 5, 3, 2, 6] + R[1, 4, 5, 3, 6, 2] + R[1, 4, 5, 6, 2, 3] +
 R[1, 4, 5, 6, 3, 2] + R[1, 4, 6, 2, 3, 5] + R[1, 4, 6, 2, 5, 3] + R[1, 4, 6, 3, 2, 5] + R[1, 4, 6, 3, 5, 2] +
 R[1, 4, 6, 5, 2, 3] + R[1, 4, 6, 5, 3, 2] + R[1, 5, 2, 3, 4, 6] + R[1, 5, 2, 3, 6, 4] + R[1, 5, 2, 4, 3, 6] +
 R[1, 5, 2, 4, 6, 3] + R[1, 5, 2, 6, 3, 4] + R[1, 5, 2, 6, 4, 3] + R[1, 5, 3, 2, 4, 6] + R[1, 5, 3, 2, 6, 4] +
 R[1, 5, 3, 4, 2, 6] + R[1, 5, 3, 4, 6, 2] + R[1, 5, 3, 6, 2, 4] + R[1, 5, 3, 6, 4, 2] + R[1, 5, 4, 2, 3, 6] +
 R[1, 5, 4, 2, 6, 3] + R[1, 5, 4, 3, 2, 6] + R[1, 5, 4, 3, 6, 2] + R[1, 5, 4, 6, 2, 3] + R[1, 5, 4, 6, 3, 2] +
 R[1, 5, 6, 2, 3, 4] + R[1, 5, 6, 2, 4, 3] + R[1, 5, 6, 3, 2, 4] + R[1, 5, 6, 3, 4, 2] + R[1, 5, 6, 4, 2, 3] +
 R[1, 5, 6, 4, 3, 2] + R[1, 6, 2, 3, 4, 5] + R[1, 6, 2, 3, 5, 4] + R[1, 6, 2, 4, 3, 5] + R[1, 6, 2, 4, 5, 3] +
 R[1, 6, 2, 5, 3, 4] + R[1, 6, 2, 5, 4, 3] + R[1, 6, 3, 2, 4, 5] + R[1, 6, 3, 2, 5, 4] + R[1, 6, 3, 4, 2, 5] +
 R[1, 6, 3, 4, 5, 2] + R[1, 6, 3, 5, 2, 4] + R[1, 6, 3, 5, 4, 2] + R[1, 6, 4, 2, 3, 5] + R[1, 6, 4, 2, 5, 3] +
 R[1, 6, 4, 3, 2, 5] + R[1, 6, 4, 3, 5, 2] + R[1, 6, 4, 5, 2, 3] + R[1, 6, 4, 5, 3, 2] + R[1, 6, 5, 2, 3, 4] +
 R[1, 6, 5, 2, 4, 3] + R[1, 6, 5, 3, 2, 4] + R[1, 6, 5, 3, 4, 2] + R[1, 6, 5, 4, 2, 3] + R[1, 6, 5, 4, 3, 2];
```

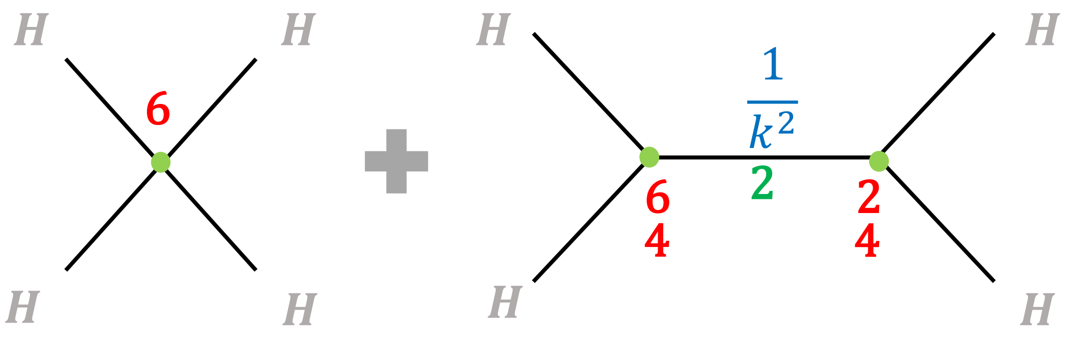
Field theory & Feynman Diag

4 fields & 6 Derivatives

$$\mathbf{S}^{(0)} = -\frac{2}{\kappa^2}\int d^Dx\, e^{-2\phi}\sqrt{-G}\left(R + 4\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{12}H^2\right)$$

$$\begin{aligned}\mathbf{S}^{(1)}_M = -\frac{2\alpha' a_1}{\kappa^2}\int d^Dx\sqrt{-G}e^{-2\Phi}\Big[&R^2_{GB} + \frac{1}{24}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}H_{\beta\delta}{}^{\varepsilon}H_{\gamma\epsilon\varepsilon} - \frac{1}{8}H_{\alpha\beta}{}^{\delta}H^{\alpha\beta\gamma}H_\gamma{}^{\epsilon\varepsilon}H_{\delta\epsilon\varepsilon}\\ &+\frac{1}{144}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}H_{\delta\epsilon\varepsilon}H^{\delta\epsilon\varepsilon} + H_\alpha{}^{\gamma\delta}H_{\beta\gamma\delta}R^{\alpha\beta} - \frac{1}{6}H_{\alpha\beta\gamma}H^{\alpha\beta\gamma}R - \frac{1}{2}H_\alpha{}^{\delta\epsilon}H^{\alpha\beta\gamma}R_{\beta\gamma\delta\epsilon}\\ &-\frac{2}{3}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\nabla^\alpha\Phi + \frac{2}{3}H_{\beta\gamma\delta}H^{\beta\gamma\delta}\nabla_\alpha\Phi\nabla^\alpha\Phi + 8R\nabla_\alpha\Phi\nabla^\alpha\Phi - 16R_{\alpha\beta}\nabla^\alpha\Phi\nabla^\beta\Phi\\ &+16\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\nabla^\beta\Phi - 16\nabla_\alpha\Phi\nabla^\alpha\Phi\nabla_\beta\Phi\nabla^\beta\Phi + 2H_\alpha{}^{\gamma\delta}H_{\beta\gamma\delta}\nabla^\beta\nabla^\alpha\Phi\Big],\end{aligned}$$

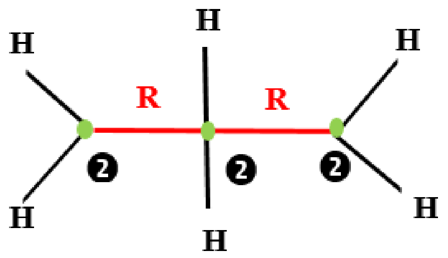
$$\begin{aligned}\mathbf{S}^{(2)B}_M = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{26}x\sqrt{-G}e^{-2\Phi}\Big[&-\frac{4}{3}R_\alpha{}^\kappa{}_\gamma{}^\lambda R^{\alpha\beta\gamma\theta}R_{\beta\lambda\theta\kappa} + \frac{4}{3}R_{\alpha\beta}{}^{\kappa\lambda}R^{\alpha\beta\gamma\theta}R_{\gamma\kappa\theta\lambda} - \frac{1}{12}H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}H_{\beta\theta}{}^\lambda H_\gamma{}^{\mu\nu}H_{\kappa\mu}{}^\tau H_{\lambda\nu\tau}\\ &+\frac{1}{4}H_{\alpha\beta}{}^\theta H^{\alpha\beta\gamma}H_\gamma{}^{\kappa\lambda}H_\theta{}^{\mu\nu}H_{\kappa\mu}{}^\tau H_{\lambda\nu\tau} + \frac{1}{48}H_{\alpha\beta}{}^\theta H^{\alpha\beta\gamma}H_\gamma{}^{\kappa\lambda}H_\theta{}^{\mu\nu}H_{\kappa\lambda}{}^\tau H_{\mu\nu\tau} - 2H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta\theta}{}^{\lambda\mu}R_{\gamma\lambda\kappa\mu} - H_{\alpha\beta}{}^\theta H^{\alpha\beta\gamma}R_\gamma{}^{\kappa\lambda\mu}R_{\theta\lambda\kappa\mu}\\ &+2H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}R_{\alpha\beta\theta}{}^\mu R_{\gamma\mu\kappa\lambda} - 2H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}R_\beta{}^\lambda{}_\gamma{}^\mu R_{\theta\lambda\kappa\mu} + \frac{1}{4}H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}\nabla_\gamma H_{\kappa\lambda\mu}\nabla_\theta H_{\alpha\beta}{}^\mu + \frac{1}{2}H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_\kappa H_{\theta\lambda\mu}\nabla^\mu H_{\beta\gamma}{}^\lambda\\ &+H_\alpha{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_\mu H_{\gamma\kappa\lambda}\nabla^\mu H_{\beta\theta}{}^\lambda\Big].\end{aligned}$$



H. Gholian and M. R. Garousi, Phys. Rev. D **109**, no.8, 086007 (2024) doi:10.1103/PhysRevD.109.086007 [arXiv:2311.05207 [hep-th]].

Field theory & Feynman Diag

6 fields & 2 Derivatives



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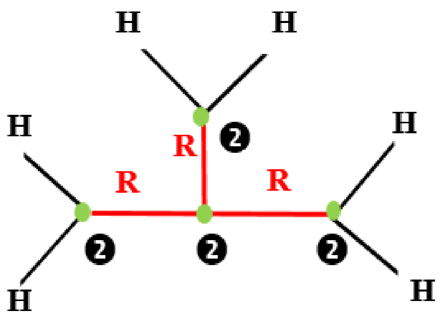

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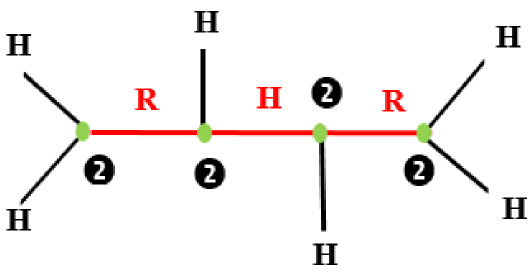

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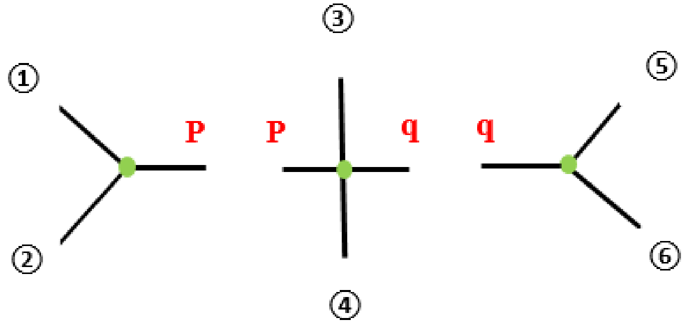


1) جایگذاری مشتق ها با ik

2) شرایط On-shell $h^{i\mu}{}_{\mu} = 0$ و $k^{i\mu} k^{i}_{\mu} = 0$

3) شرط پیمانه‌ای $k^{i\mu} \mathbb{b}^{i}{}_{\mu\nu} = 0$ و $k^{i\mu} h^{i}{}_{\mu\nu} = 0$

R[1, 2, 3, 4, 5, 6] + R[1, 2, 3, 5, 4, 6] + R[1, 2, 3, 6, 4, 5] + R[1, 2, 4, 5, 3, 6] + R[1, 2, 4, 6, 3, 5] +
R[1, 2, 5, 6, 3, 4] + R[1, 3, 2, 4, 5, 6] + R[1, 3, 2, 5, 4, 6] + R[1, 3, 2, 6, 4, 5] + R[1, 3, 4, 5, 2, 6] +
R[1, 3, 4, 6, 2, 5] + R[1, 3, 5, 6, 2, 4] + R[1, 4, 2, 3, 5, 6] + R[1, 4, 2, 5, 3, 6] + R[1, 4, 2, 6, 3, 5] +
R[1, 4, 3, 5, 2, 6] + R[1, 4, 3, 6, 2, 5] + R[1, 4, 5, 6, 3, 2] + R[1, 5, 2, 6, 3, 4] + R[1, 5, 3, 2, 4, 6] +
R[1, 5, 3, 4, 2, 6] + R[1, 5, 3, 6, 4, 2] + R[1, 5, 4, 2, 3, 6] + R[1, 5, 4, 6, 3, 2] + R[1, 6, 3, 2, 4, 5] +
R[1, 6, 3, 4, 5, 2] + R[1, 6, 3, 5, 4, 2] + R[1, 6, 4, 2, 3, 5] + R[1, 6, 4, 5, 3, 2] + R[1, 6, 5, 2, 3, 4] +
R[2, 3, 1, 4, 5, 6] + R[2, 3, 1, 5, 4, 6] + R[2, 3, 1, 6, 4, 5] + R[2, 4, 1, 3, 5, 6] + R[2, 4, 1, 5, 3, 6] +
R[2, 4, 1, 6, 3, 5] + R[2, 5, 1, 3, 4, 6] + R[2, 5, 1, 4, 3, 6] + R[2, 5, 1, 6, 3, 4] + R[2, 6, 1, 3, 4, 5] +
R[2, 6, 1, 4, 3, 5] + R[2, 6, 1, 5, 3, 4] + R[3, 4, 1, 2, 5, 6] + R[3, 5, 1, 2, 4, 6] + R[3, 6, 1, 2, 4, 5]

در نظر گرفتن فقط حالت ۱–تریسی

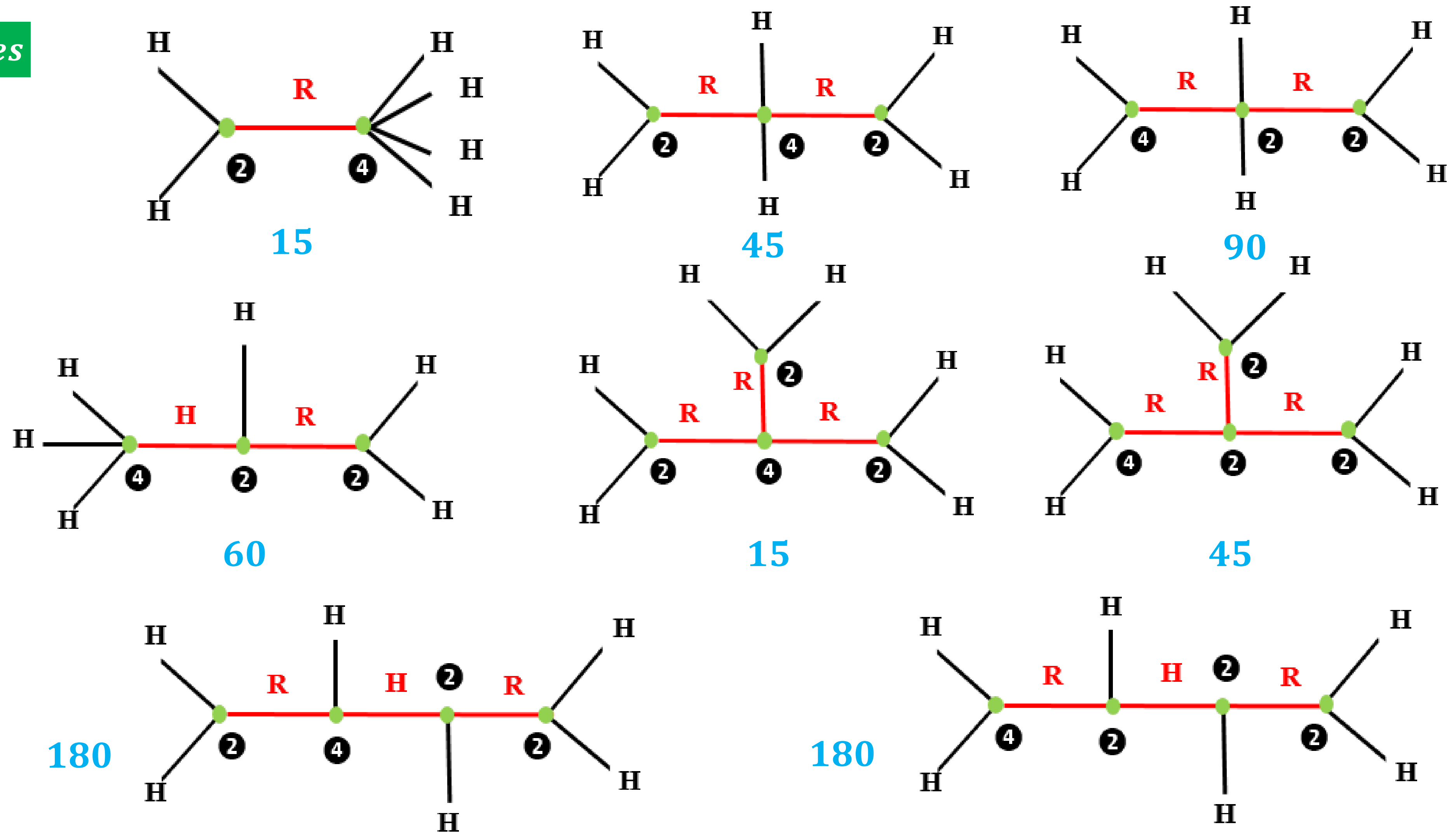
$$\mathbb{b}^1 \mathbb{b}^2 \mathbb{b}^3 \mathbb{b}^4 \mathbb{b}^5 \mathbb{b}^6$$

$$g, h, j = 0$$

$$-\frac{(a+d)\ (c+f)\ \mathbb{b}^{1AB}\ \mathbb{b}^{2}{}_{A}{}^{F}\ \mathbb{b}^{3}{}_{F}{}^{H}\ \mathbb{b}^{4}{}_{H}{}^{J}\ \mathbb{b}^{5}{}_{J}{}^{L}\ \mathbb{b}^{6}{}_{BL}}{4\, b}$$

Field theory & Feynman Diag

6 fields & 4 Derivatives



$$-\frac{(a+d)\,(c+f)\,(a-b+c+d+f)\,\mathbb{b}^{1AB}\,\mathbb{b}^{2}{}_{A}{}^{F}\,\mathbb{b}^{3}{}_{F}{}^{H}\,\mathbb{b}^{4}{}_{H}{}^{J}\,\mathbb{b}^{5}{}_{J}{}^{L}\,\mathbb{b}^{6}{}_{BL}}{8\,b}$$

Field theory & Feynman Diag

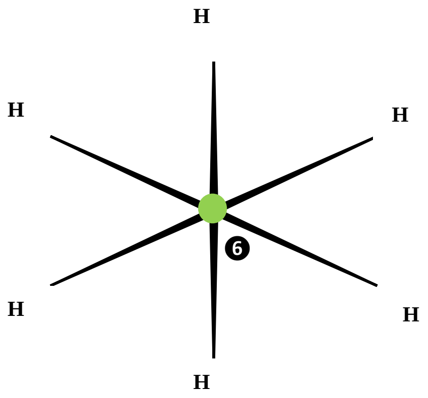
6 fields & 6 Derivatives

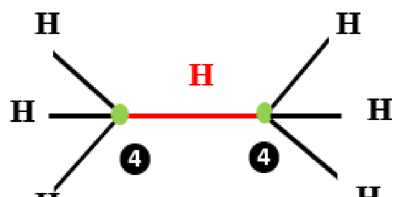
$$\mathbf{S}_M^{(2)B} = -\frac{2\alpha'^2 a_1^2}{\kappa^2}\int d^{26}x\sqrt{-G}e^{-2\Phi}\Big[-\frac{4}{3}R_{\alpha}{}^{\kappa}{}_{\gamma}{}^{\lambda}R^{\alpha\beta\gamma\theta}R_{\beta\lambda\theta\kappa}+\frac{4}{3}R_{\alpha\beta}{}^{\kappa\lambda}R^{\alpha\beta\gamma\theta}R_{\gamma\kappa\theta\lambda}-\frac{1}{12}H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}H_{\beta\theta}{}^{\lambda}H_{\gamma}{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau}$$

$$+\frac{1}{4}H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\kappa\lambda}H_{\theta}{}^{\mu\nu}H_{\kappa\mu}{}^{\tau}H_{\lambda\nu\tau}+\frac{1}{48}H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}H_{\gamma}{}^{\kappa\lambda}H_{\theta}{}^{\mu\nu}H_{\kappa\lambda}{}^{\tau}H_{\mu\nu\tau}-2H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta\theta}{}^{\lambda\mu}R_{\gamma\lambda\kappa\mu}-H_{\alpha\beta}{}^{\theta}H^{\alpha\beta\gamma}R_{\gamma}{}^{\kappa\lambda\mu}R_{\theta\lambda\kappa\mu}$$

$$+2H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}R_{\alpha\beta\theta}{}^{\mu}R_{\gamma\mu\kappa\lambda}-2H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}R_{\beta}{}^{\lambda}{}_{\gamma}{}^{\mu}R_{\theta\lambda\kappa\mu}+\frac{1}{4}H^{\alpha\beta\gamma}H^{\theta\kappa\lambda}\nabla_{\gamma}H_{\kappa\lambda\mu}\nabla_{\theta}H_{\alpha\beta}{}^{\mu}+\frac{1}{2}H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_{\kappa}H_{\theta\lambda\mu}\nabla^{\mu}H_{\beta\gamma}{}^{\lambda}$$

$$+H_{\alpha}{}^{\theta\kappa}H^{\alpha\beta\gamma}\nabla_{\mu}H_{\gamma\kappa\lambda}\nabla^{\mu}H_{\beta\theta}{}^{\lambda}\Big].$$





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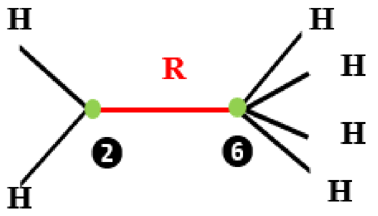

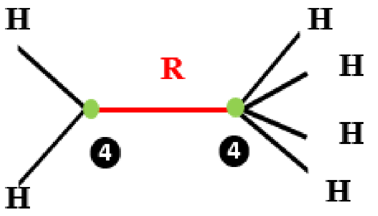

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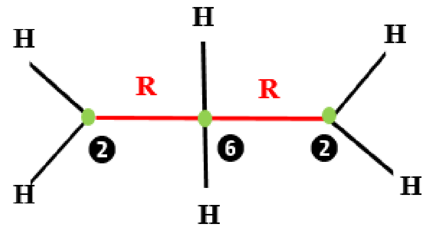

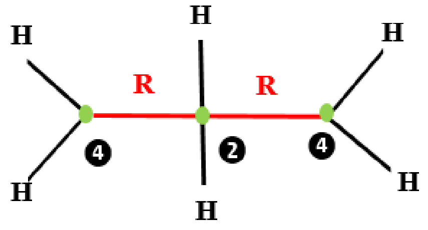

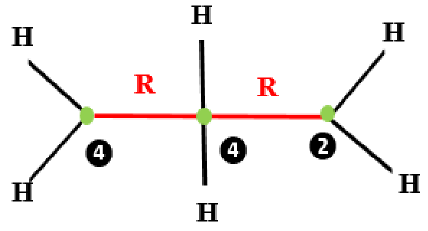

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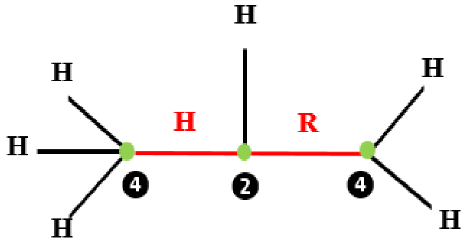

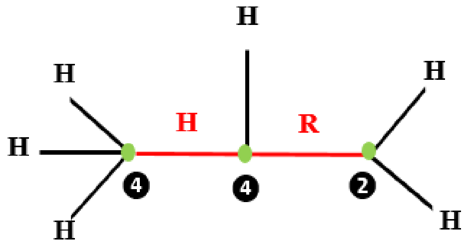

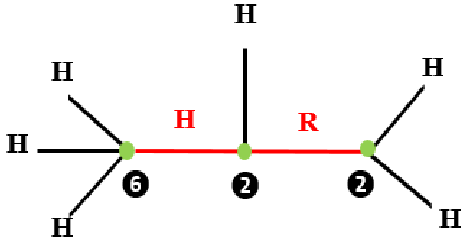

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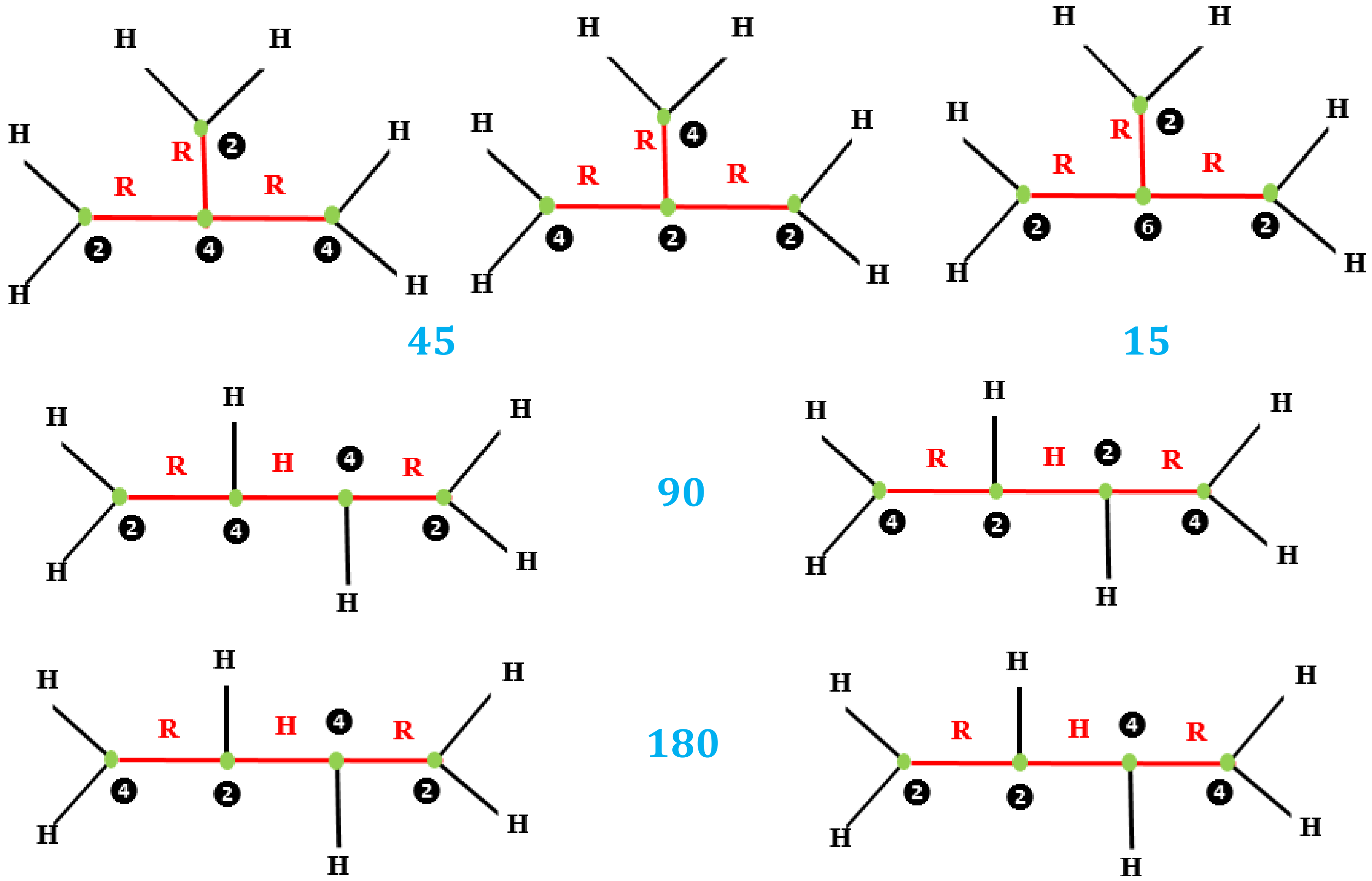
Field theory & Feynman Diag

6 fields & 6 Derivatives



$$\frac{2\,(a+d)\,(c+f)\,\left(a^2+c^2+c\,d+d^2+c\,f+d\,f+f^2+a\,(-b+c+d+f)-b\,(c+d+e+f)\right)\,\mathbb{b}^{1AB}\,\mathbb{b}^{2}{}_{A}{}^{F}\,\mathbb{b}^{3}{}_{F}{}^{H}\,\mathbb{b}^{4}{}_{H}{}^{J}\,\mathbb{b}^{5}{}_{J}{}^{L}\,\mathbb{b}^{6}{}_{BL}}{b}$$

Conclusion

1) به دست آوردن کنشی کاربردی با کمترین جملات ممکن برای اولین بار در تئوری بوزونی و هتراتیک

2) محاسبه بسط S-matrix پراکندگی ۶ ریسمان بوزونی بسته

3) پیدا کردن روشی کاربردی برای بسط فیزیکی تابع triple hypergeometric

4) تایید همخوان بودن بسط S-matrix پراکندگی ۶ ریسمان بوزونی با محسابات نمودار فایمن نظریه میدان تا مرتبه ۴ مشتق