Natural Selection and Random Matrix Theory

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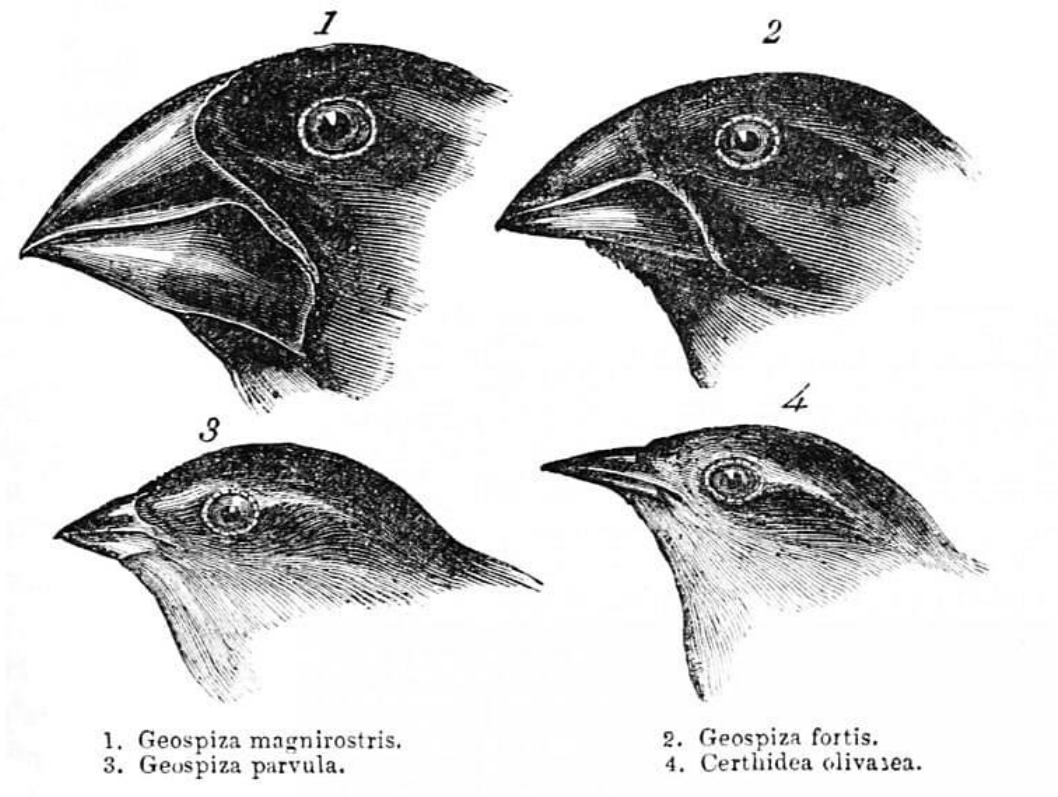
0. Final Result !!!

$$\text{Natural Selection} \equiv \text{GWW Phase Transition}$$

In this presentation, we try to explain **Why?!**

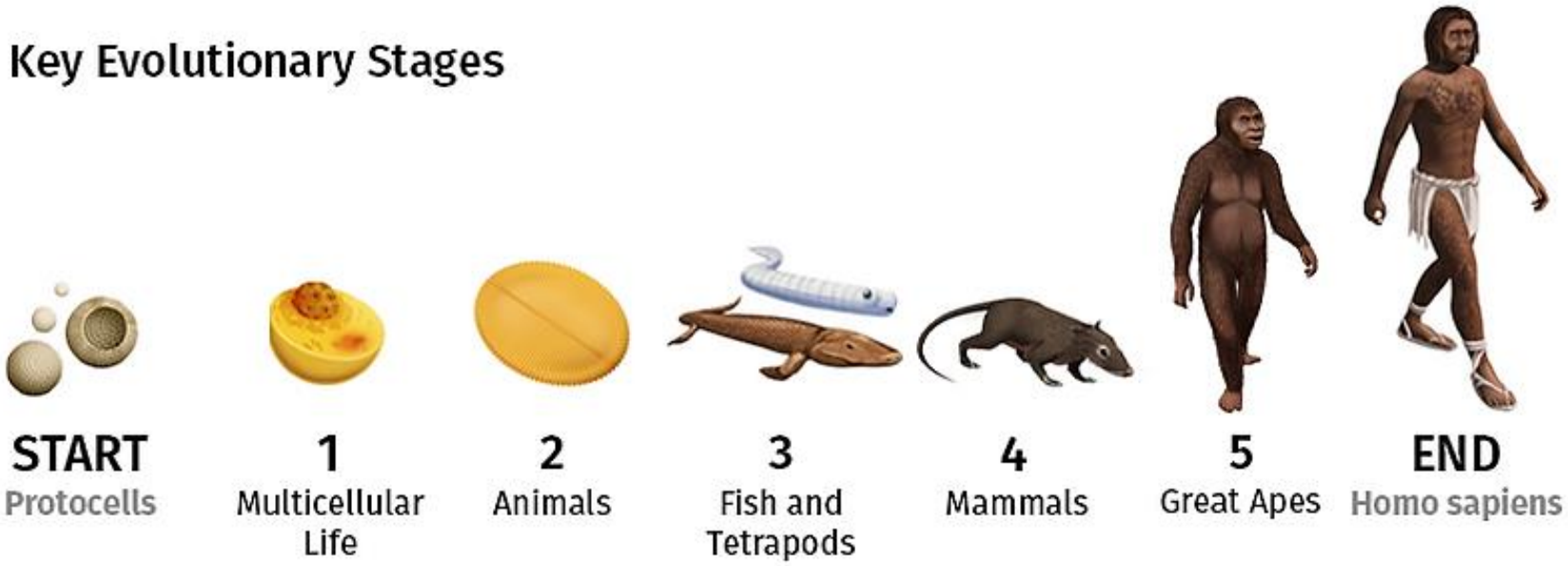
1. Introduction

- Genetic evolution occurs through changes in the heritable characteristics of a population over *successive generations*.
- Individuals with traits that are advantageous for **survival** and **reproduction** are more likely to pass those traits to the next generation.



1. Introduction

- Over time, these advantageous traits become more common in the population, leading to evolutionary changes
- Result of the accumulation of genetic changes that are inherited over time
- Evolution can be observed as changes in the ***ensemble*** of genes



1. Introduction

$$\textit{Evolution} = \sum \textit{Mutation} \rightarrow \textit{Natural Selection}$$

- It happens when a creature undergoes unexpected natural stresses and then it naturally selects the more stable state.
- It strongly correlates with external parameters such as pressure, temperature, food resources, cosmic rays, etc.
- It also depends on the occurrence of natural phenomena (asteroids, etc.)

1. Introduction

On what scale does the evolution happen?!

First, We discuss the subject on a scale where the quantum dynamic is dominant.

If the evolution happens quantum mechanically, then the phase space is the space of possible states or Hilbert space which contains finite possible states to be observed by the observer: ***Quantum Darwinism*** [Zurek '09]

This scenario points out that a cell can choose between an infinite number of possible states with random distribution each with a specific probability weight.

Either in classical or quantum regimes *(will be discussed later):*

One can consider evolution as a set of random states to be chosen from a Hermitian random matrix ensemble (Random Matrix Theory)

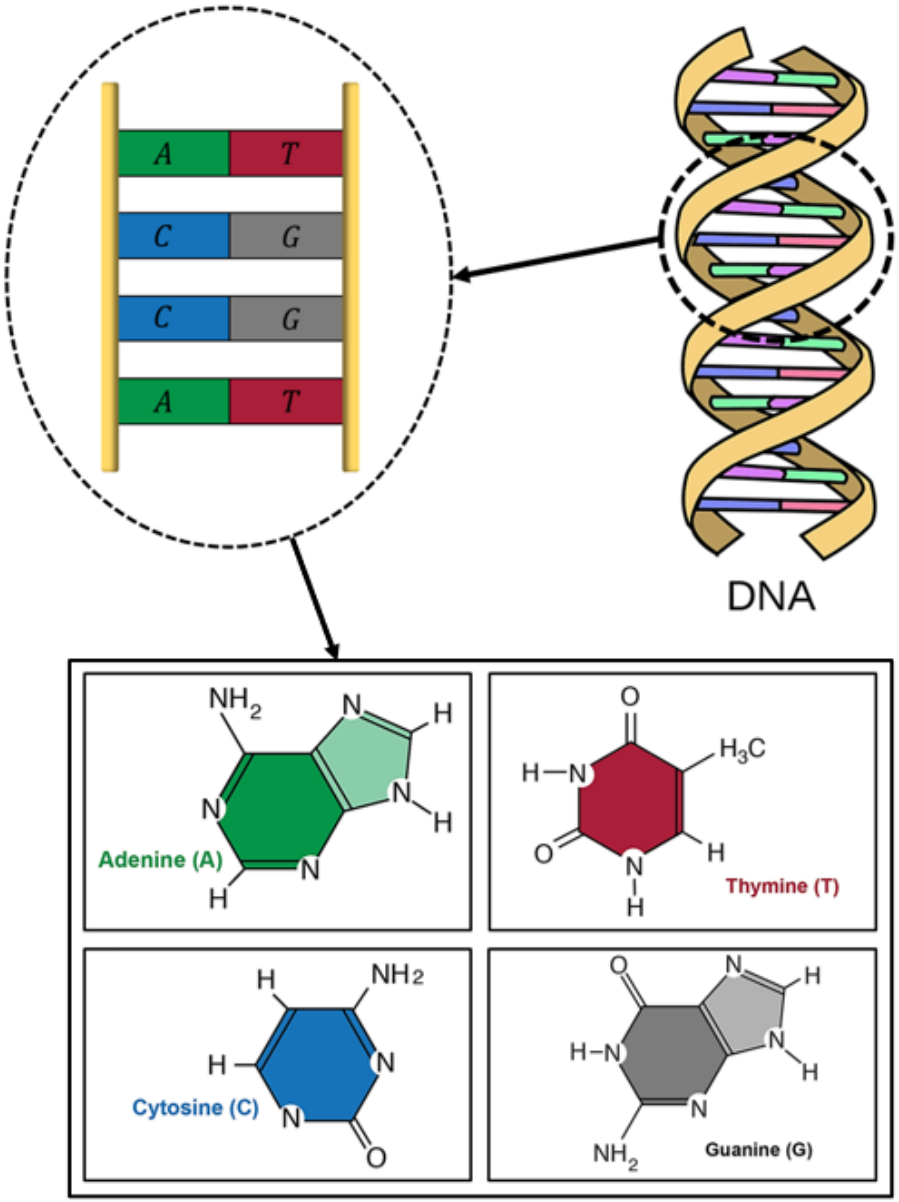
2. RMT, Phase Transition and Evolution

* ***A single DNA is constructed of:***

Two sugar-phosphate backbones

Four nucleotides:
Adenine (A), Thymine (T),
Cytosine (C), and Guanine (G)

Two base pair bindings AT and GC

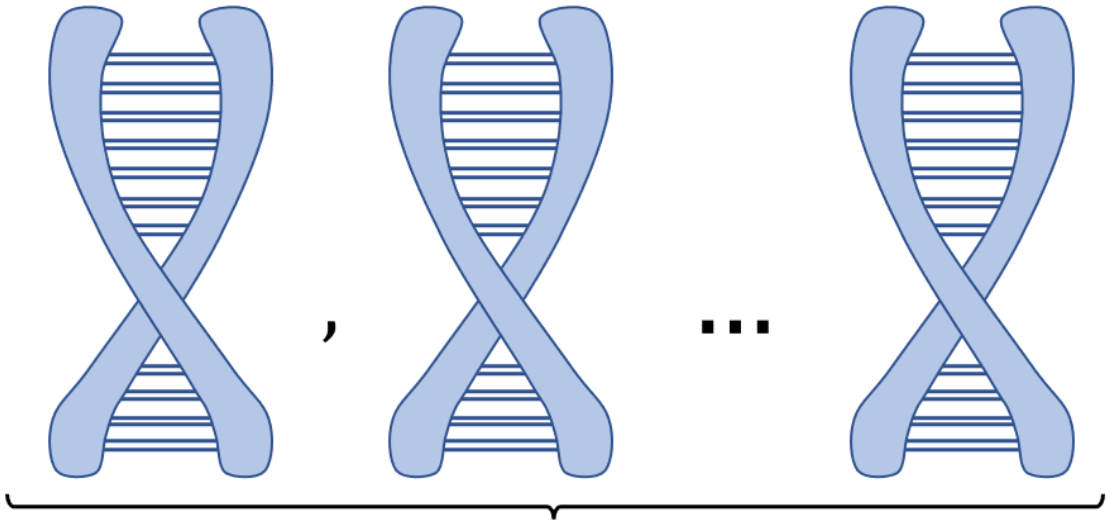


- GC base pairs are held together by **three** hydrogen bonds, compared to **two** AT base pairs, making GC pairs more thermally stable.

$$F = -\nabla U$$

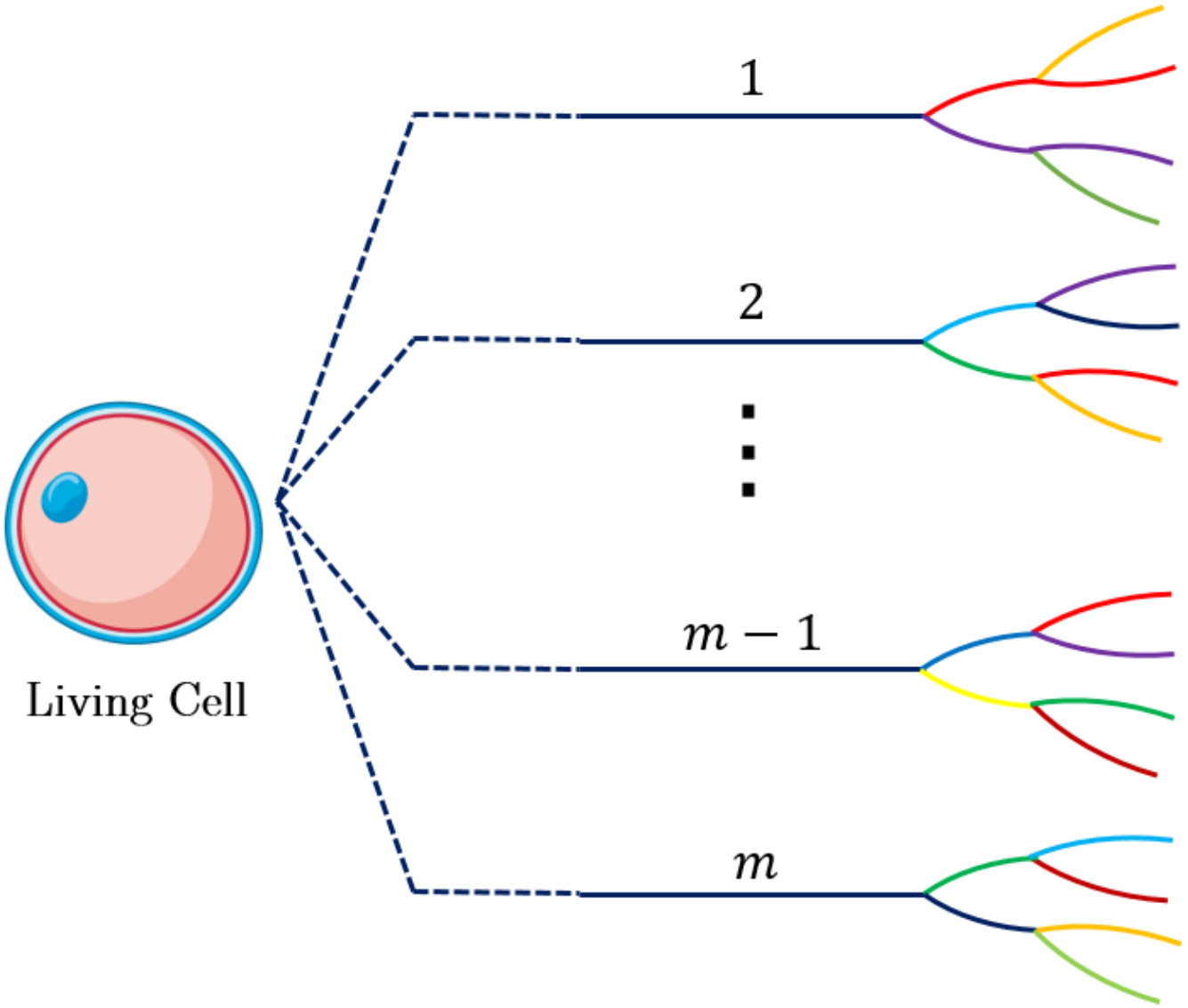
2. RMT, Phase Transition and Evolution

$$diag(H) = \begin{pmatrix} \lambda_{AT}^{(1)} & 0 & 0 & \dots \\ 0 & \lambda_{GC}^{(2)} & 0 & \dots \\ 0 & 0 & \lambda_{GC}^{(3)} & \dots \\ 0 & 0 & 0 & \dots \end{pmatrix}_{L \times L} \qquad L \to \infty$$



An ensemble of n copies of DNA

2. RMT, Phase Transition and Evolution



2.1. Gross-Witten-Wadia Phase Transition

When does genetic evolution occur in the decomposition process?

*** From a biological point of view:**

$$A \xrightarrow{\text{exchange}} C$$

in the replication which in turn changes the features of the living cell

$A \rightarrow C$ or $T \rightarrow G$: generate a gap in the energy spectrum of the DNA to put the cell in a more stable state (since GC bounds are stronger). In turn, it slightly changes the shape of $\rho(\lambda)$.

This bridge describes a variation between two phases:

weak (more AT) and strong (more GC)

2.1. Gross-Witten-Wadia Phase Transition

* Any random complex interacting system will remain stable ***if*** **[May '72]:**

$$\lambda_{max} < (g)^{-1}$$

There is a critical value g_c in which the system would experience a phase transition

- A similar phase transition occurs in 2d lattice gauge theory (*when* $\boldsymbol{L \to \infty}$, $\boldsymbol{g \to g_c}$) which is known as the **Gross-Witten-Wadia (GWW)** phase transition in QCD

- Using the ***steepest-descent*** method **[Brezin, Itzykson, Parisi, Zuber '78]**:

They showed for unitary matrices, the behavior of $\rho(\lambda)$ for weak and strong coupling regimes is described by ***different analytical functions***

They have also conjectured such a phase transition for the ***4-dimensional*** *case*

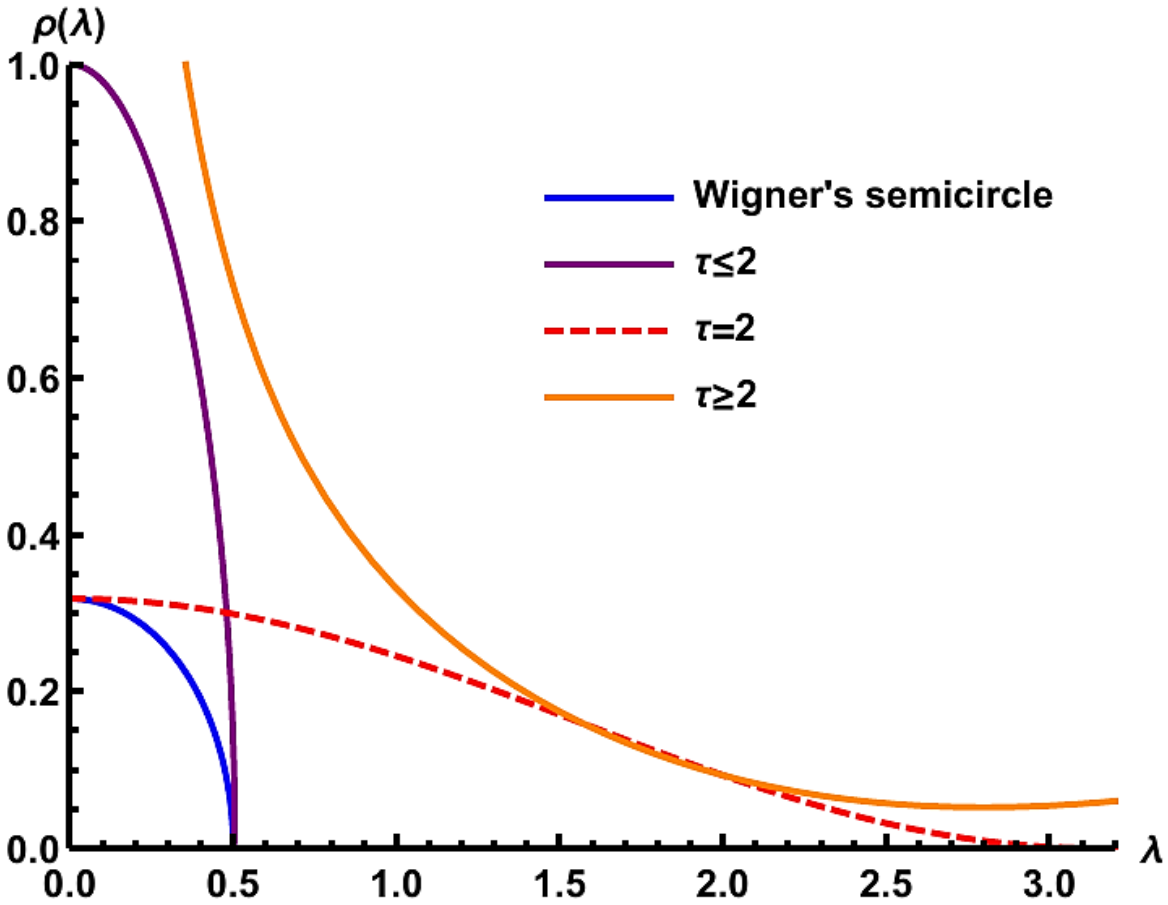
2.1. Gross-Witten-Wadia Phase Transition

$$\rho(\lambda) = \begin{cases} \frac{2}{\pi\tau} cos\frac{\lambda}{2}\left(\frac{\tau}{2} - sin^2(\frac{\lambda}{2})\right)^{1/2}, & \tau \leq 2 \\ \frac{1}{2\pi}\left(1 + cos\lambda\right), & \tau = 2 \\ \frac{1}{2\pi}\left(1 + \frac{2}{\lambda} cos\lambda\right), & \tau \geq 2 \end{cases}$$

$$\tau = g^2 L$$

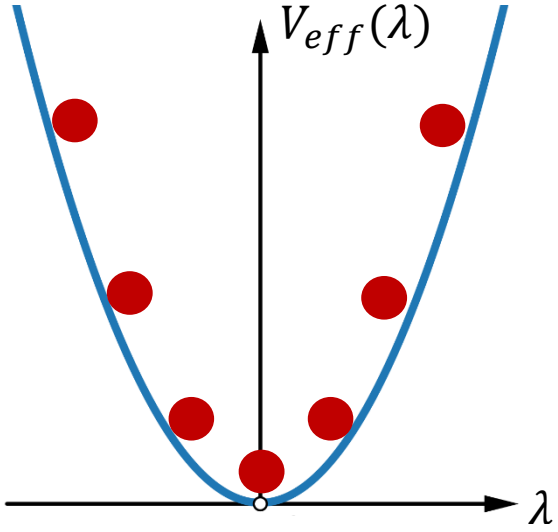
For $|\lambda| \leq \sqrt{2\tau}$ and $\tau \to 0$

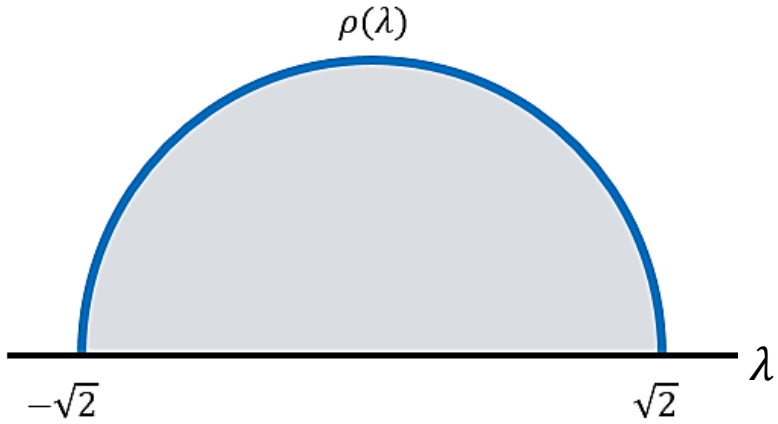
$$\rho(\lambda) \simeq \frac{1}{\pi}\sqrt{1-\lambda^2}$$



2.2. Tracy-Widom Distribution

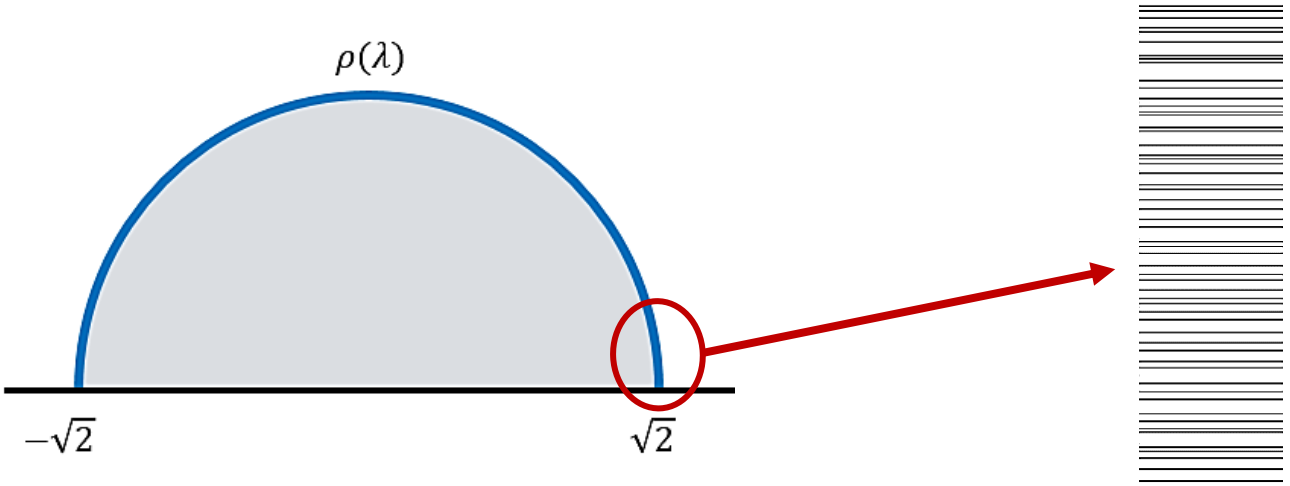
- *The force between eigenvalues can differ by changing g and that will cause a slight deviation to the **effective potential** where the eigenvalues condensed:*





2.1. Gross-Witten-Wadia Phase Transition

Very small coupling: *the distribution will be as random as possible (sharp edge)*



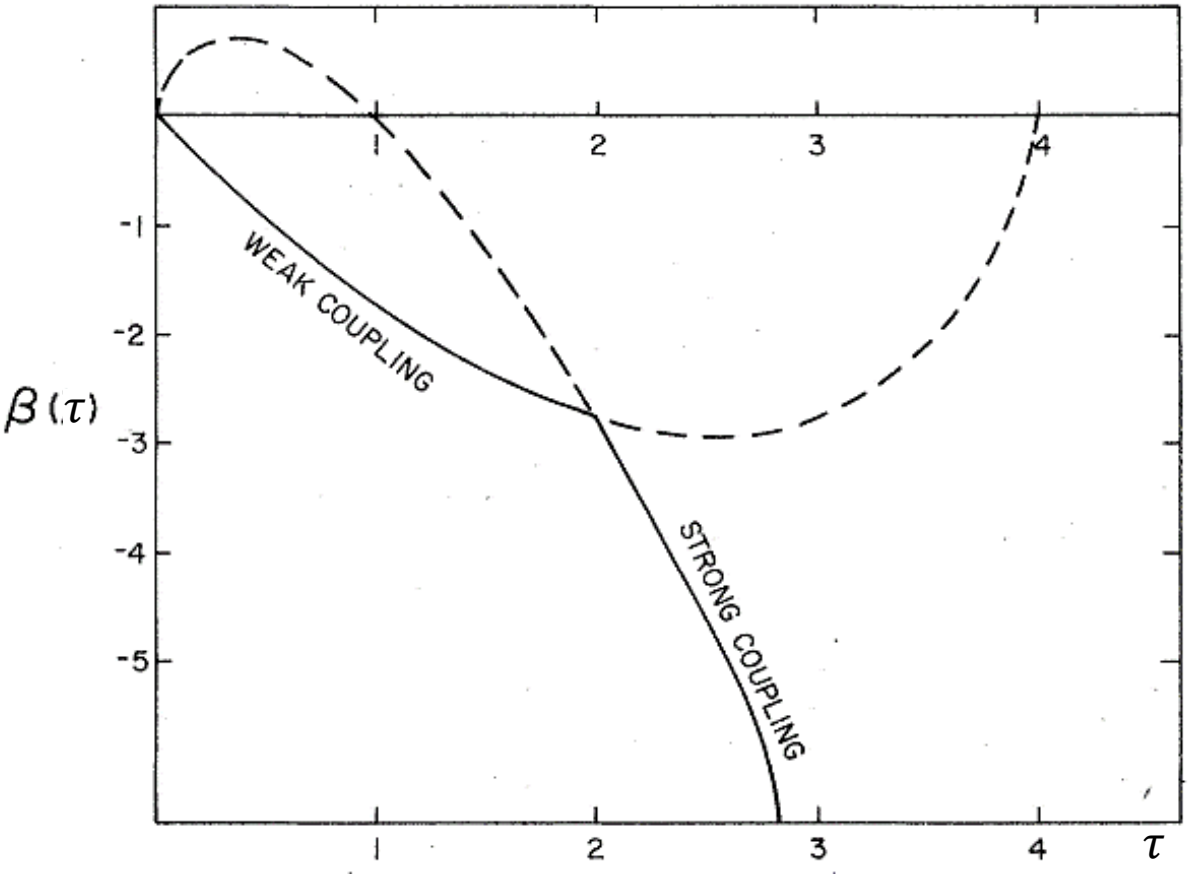
PT takes place exactly when the eigenvalues completely occupy the entire circle

Very large coupling: *a repulsion among the eigenvalues, leading to a uniform distribution*

2.1. Gross-Witten-Wadia Phase Transition

- *A string model [‘t Hooft ‘74] describing the large L perturbative expansion of QCD could also arise within the lattice framework*

- *Adjusting the lattice spacing $\boldsymbol{a}$ and the coupling constant such that the **string tension** remains constant would help to define an **effective coupling***

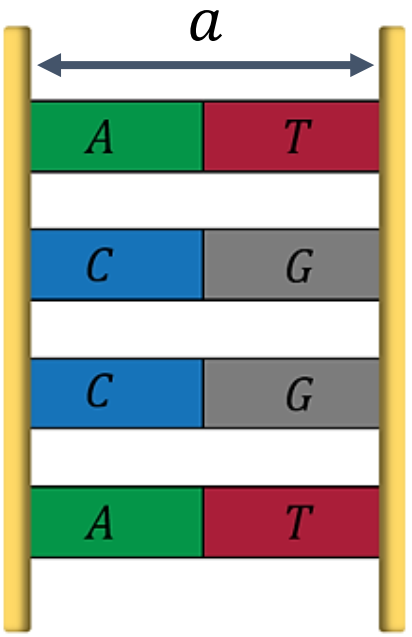


2.1. Gross-Witten-Wadia Phase Transition

- β-function of the theory does not vanish at $\tau = 2$
- PT is of a higher order **than two**, otherwise the string tension must vanish
- It's of **3rd order.** 3rd derivative of the free energy has a discontinuity at $\tau = 2$

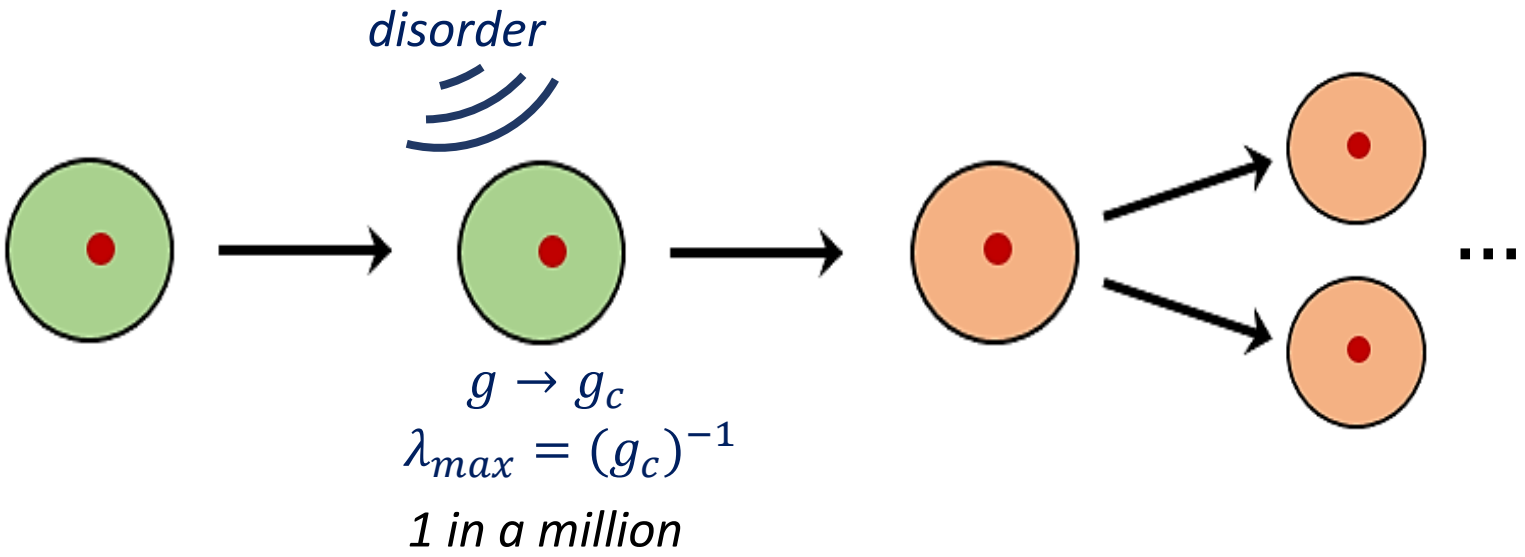
We consider each DNA as a 2d lattice with random coulomb-forces, analogous to a $U(L)$ lattice gauge theory. The bindings are considered as strings with tension κ, such that:

$$\kappa[\bar{a}, g(\bar{a})] = \kappa$$



2.1. Gross-Witten-Wadia Phase Transition

In the fission, DNA may encounter disorders: $L \to \infty$



The coupling constant does not always change,
It changes in a way to preserve the tension

The nucleotides will encounter such instabilities and decide to choose a stronger structure for the next steps of the replication process

2.1. Gross-Witten-Wadia Phase Transition

- ***Some important points***

- At the moment of decomposition, one specific binding should not necessarily have the exact value of previous energy and it gets picked randomly in later steps

- Energy of each bond λ_{AT} and λ_{GC} can not exceed a *min* and a *max* value

- If so, the binding would be destroyed in one part of the DNA and the tension vanishes

- If $\#\lambda_{GC} = \#\lambda_{AT}$ (with $\lambda_{GC} > \lambda_{AT}$), changing $A \rightarrow C$ will change the distribution from the equilibrium point

2.1. Gross-Witten-Wadia Phase Transition

- ***Some important points***

- Therefore, the interaction between AT and GC will turn into a new regime with a less coupling constant since $\lambda \sim g^{-1}$ (more randomness)

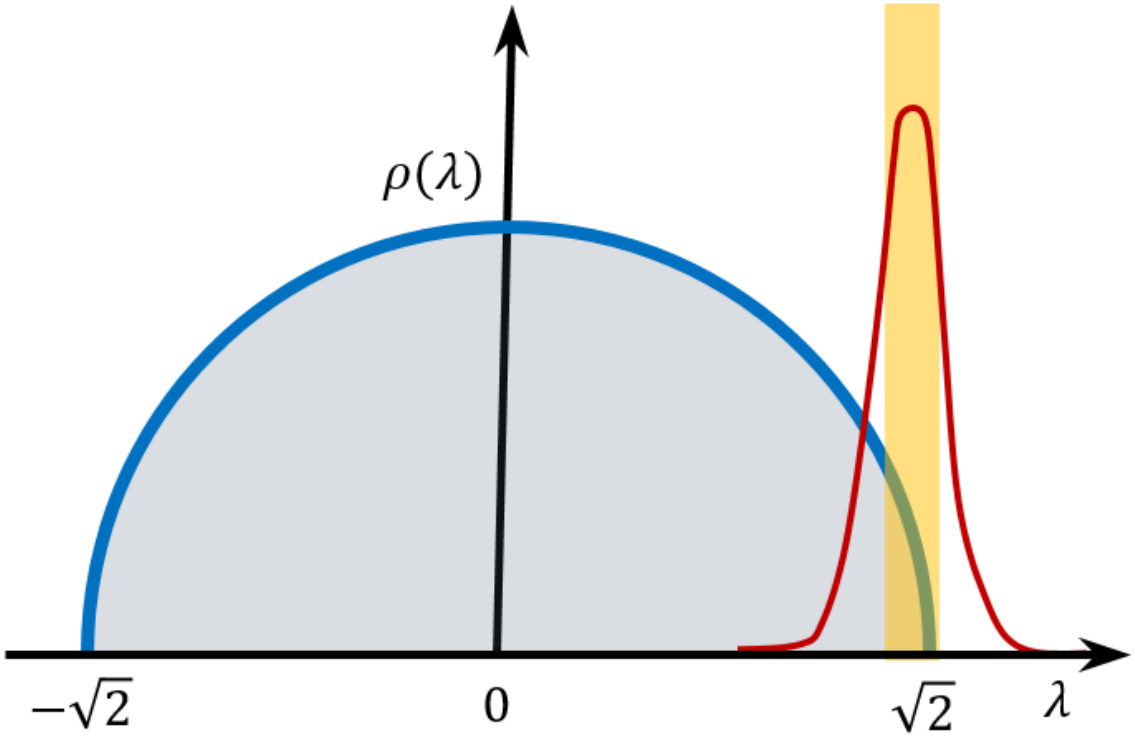
- We tune the coupling such that the structure remains stable

- This is similar to the GWW phase transition from weak to strong coupling phase which can turn the DNA into a more stable regime

- Since this phase transition occurs randomly in the fission process, it is somehow equivalent to natural selection.

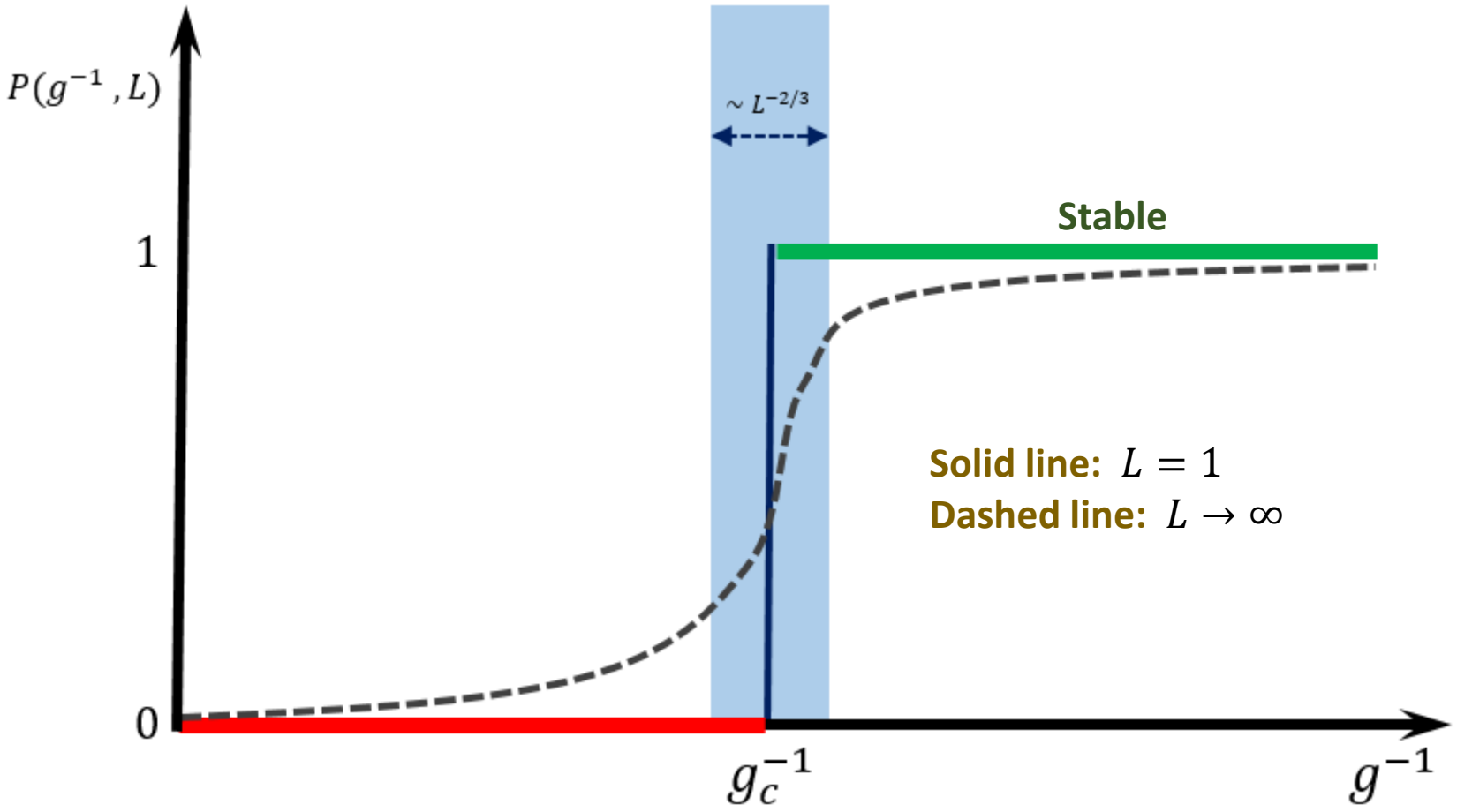
2.2. Tracy-Widom Distribution

- It's the distribution of the maximum eigenvalue λ_{max}
- Applied close to the edge of the semi-circle
- Helps us to find the ***crossover functions*** between weak and strong phases in the double-scaling limit [Majumdar, Schehr '13]



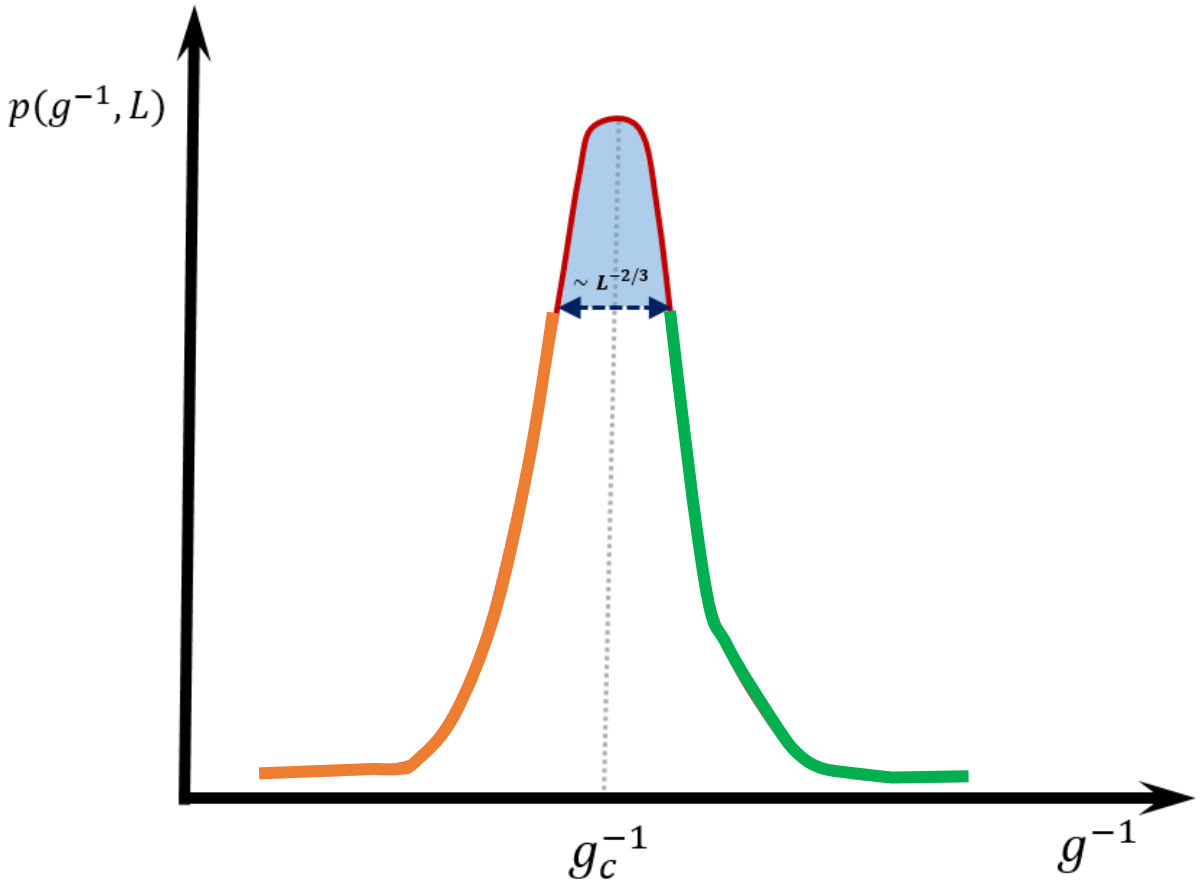
2.2. Tracy-Widom Distribution

- Phase transition can occur on a gap of order $L^{-2/3}$ [Majumdar, Schehr '13]



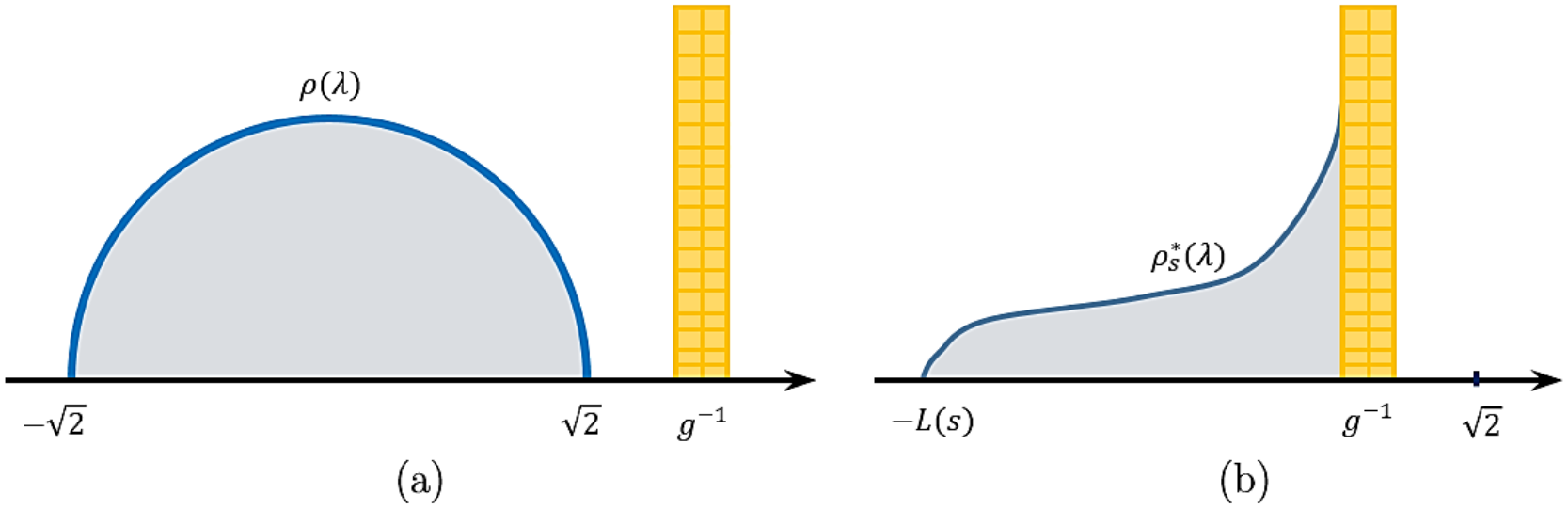
2.2. Tracy-Widom Distribution

- *The hill is described by TW distribution*
- *The tails are given by certain functions that satisfy Painlevé II equations*
- GWW phase transition separates the left and the right tail



2.2. Tracy-Widom Distribution

- *Recall that for any system we had the stability condition:* $\lambda_{max} < (g)^{-1}$
- *Therefore* g^{-1} *can be seen as a potential wall (sliding on the real axis energy line)*
- *When the coupling constant gets smaller,* g^{-1} *gets bigger and the potential wall will move away from the semi-circle (*$Energy\ cost\ = Energy\ of\ A \rightarrow C$*)*



- *Out of the semi-circle: no phase transition will appear*
- *Hitting the edge: Affect the eigenvalues and change the Wigner distribution*
- *The distribution changes for a short time and then we get the primary form*

3. Conclusion and Remarks

Interaction changes randomly in each time step:
this can transform the coupling constant to a new value which in a specific step, ***could be the critical value*** *where the phase transition occurs*

GWW phase transition leads to a change in specific energy eigenvalue which must be preserved in the whole process

The change in one λ *directly changes the other eigenvalues due to the correlation and the repulsion force between each pair of eigenvalues*

3. Conclusion and Remarks

* The cell selects a state (among many) in which the base pair bindings take random and independent energy values in each time step chosen from GUE

* Probe exact statistical parameters of the cell for a long period (by TW)

* The change in DNA bindings can directly change g, leading to a GWW phase transition from unstable to stable regime (similar to NS)

* Free energy of DNA in large L → comparing with the string model

* Connecting mechanisms inside the cell to the mathematical developments of RMT → tracking the number of lost circles and drawing a historical map of different evolutions

* Analyzing the spectrum in the GWW phase transition → In what exact statistical circumstances the cells have more chance to survive?

Thank You!