
بسم الله الرحمن الرحیم

Resolving Angular Momentum Flux Non-Invariance in 4D Asymptotically Flat Spacetimes

Reza Javadinezhad

NYU

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Outline

1. Review of the BMS group

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2. Angular momentum problem

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5. Concluding remarks and future directions

Introduction to BMS group

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$$ds^2 = -dt^2 + dx^2.$$

2. Massless particles move on null curves, which means that for their description a coordinate system with a null coordinate is more suitable, we define retarded time $u = t - x$.
3. The metric in (u, x) coordinate is,

$$ds^2 = -du^2 - 2dudx.$$

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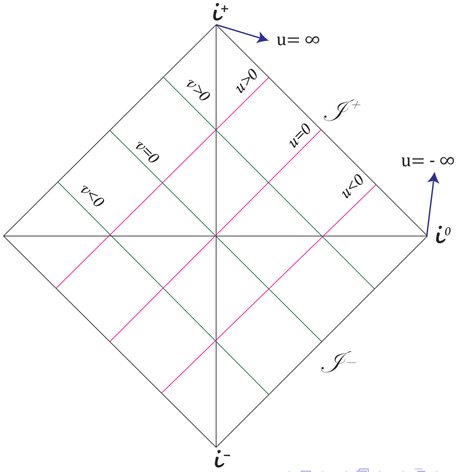
3. Null infinity $\mathscr{I}^+$ is reached when either u or v is kept constant while $t \to \infty$.
4. Notice the difference between future and past infinities.

Introduction to BMS group

We can use the freedom of the choice of coordinates (diffeomorphism) to represent the infinities at finite coordinates. This is called Penrose diagram.



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The region of spacetime where $x \to \infty$, but the retarded time u is finite.

3. Since in the asymptotic region the space is approximately flat, the dynamic of the system will be much simpler. In this region we can easily identify dynamical degrees of freedom.

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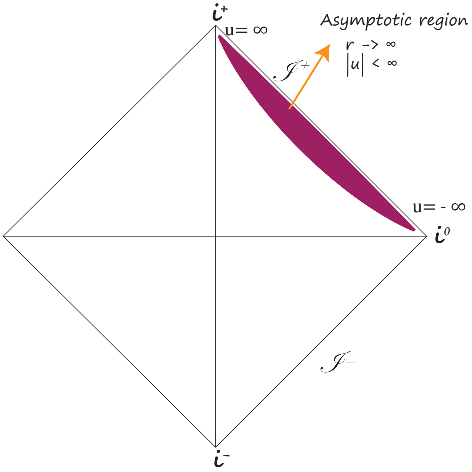

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2. Naturally we are only interested in the metrics that are solutions of the Einstein equations in the asymptotic region.

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}.$$

3. Since the gauge freedom in the general relativity is the freedom in the choice of coordinates, to eliminate the gauge redundancy we use the Bondi gauge,

$$g_{rr} = g_{r\theta} = g_{r\phi} = 0.$$

Introduction to BMS group

1. Then the set of asymptotically flat metrics looks like $(x^A = (\theta, \phi))$,

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\begin{aligned}
ds^2 = & -du^2 - 2dudr + r^2 q_{AB} dx^A dx^B + \frac{2m(u,x)}{r} du^2 \\
& + rC_{AB}(u,x) dx^A dx^B + D^B C_{AB} du dx^A \\
& \frac{C_{AB}C^{AB}}{16r^2} dudr + \left(\frac{4N_A(u,x)}{3} - \frac{1}{8} D_A (C_{CB}C^{CB})\right) \frac{dudx^A}{r} \\
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2. The function $m(u, x^A)$ is called Bondi mass aspect and includes the information about the mass multipoles of the spacetime.
3. $N_A(u, x^A)$ is the angular momentum aspect, we use it to read the angular momentum of the spacetime.
4. C_{AB} is the shear tensor and it is the main subject of this talk.

Introduction to BMS group

As an example, the Kerr metric corresponds to $C_{AB} = 0$, $m(u, x^A) = M$ and $N_A(u, x^A) = J$. Traditional expressions for the energy and angular momentum are

$$M(u) = \int_{S^2} f(x)\; m(u, x)\sqrt{q}dx^2,$$
$$J(u) = \int_{S^2} Y^A N_A(u, x)\sqrt{q}dx^2,$$

where Y^A are the generators of Lorentz group. One can check that in this case these formula give the correct answer!

Angular momentum

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3. BMS (Bondi-Metzner-Sachs) transformations are defined as such coordinate transformations.
4. The explicit form of the generators is

Introduction to BMS group

1. Generators ξ of the BMS group are

$$\xi^u = f(x) + \frac{u}{2} D.Y,$$
$$\xi^A = Y^A,$$
$$\xi^r = -\frac{r}{2} D.Y + \frac{1}{2} D^2 \xi^u.$$

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2. $f(x)$ is an arbitrary function on the sphere and it is called a "supertranslation", because it is an angle dependent time translation. Y^A are the conformal killing vectors on the sphere $D_A Y_B + D_B Y_A = q_{AB} D.Y$.

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3. Under a supertranslation "shear tensor" C_{AB}, changes $C_{AB} - (2D_A D_B - q_{AB}\Delta)f$.

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then this is a spatial translation $\vec{r} \to \vec{r} + \vec{C}$.
3. The modes Y_l^m with $l > 1$ are the new type of allowed translations (aka proper supertranslations).

Introduction to BMS group

BMS group

The symmetry group of the asymptotically flat spacetimes. BMS group is an infinite dimensional group constructed from the Lorentz group and the supertranslations.

$$[\xi^{(Y,f)}, \xi^{(Y',f')}] = \xi^{([Y,Y'], f\,[Y] - f'\,[Y'])},$$

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A crucial observation is that rotations and supertranslations do not commute. This is normal and expected for ordinary translations but not for supertrasnaltions!

Introduction to Asymptotic symmetry group

1. $C_{AB}(u, x^A)$, $N_A(u, x^A)$ and $m(u, x^A)$ carry all the information, but they are not all independent:

$$\begin{aligned}\partial_u m =& -\frac{1}{8}N_{AB}N^{AB} + \frac{1}{4}D_A D_B N^{AB},\\ \partial_u N_A =& D_A m + \frac{1}{16}D_A(N_{BC}C^{BC}) - \frac{1}{4}N^{BC}D_A C_{BC}\\ &+ \frac{1}{4}D_B D_A D_C C^{BC} - \frac{1}{4}D_B D^B D^C C_{AC}\\ &- \frac{1}{4}D_B(C^{BC}N_{AC} - N^{BC}C_{AC}),\end{aligned}$$

where $N_{AB} = \partial_u C_{AB}$ is called Bondi news tensor.

Boundary graviton

1. If there is no incoming radiation then generally the shear tensor is not zero in the past, it always can be written as

$$C_{AB}(-\infty, x) = (-2D_A D_B + q_{AB}\Delta)C(x), \qquad (1)$$

   $C(x)$ is called "boundary graviton".

   $C(x)$ doesn't directly affect mass and angular momentum, so from that point of view it is an arbitrary field.

2. Under a supertranslation

$$C \to C + f.$$

Angular momentum problem

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2. This is a problem because considering two states only differing by a supertranslation they have different angular momentum while they are equivalent for the observers performing finite time experiments.
3. This problem is not merely a formal problem, blackhole mergers radiate energy and angular momentum. If not treated correctly they also suffers from this problem. This applies to other gravitational scattering problems as well. Therefore the problem is how to separate long wavelength effects from the interesting physical data in an experiment.

Solution

1. The solution is to use the electric part of the shear tensor $C(u, x^A)$ defined below.

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Invariant charge

$$Q_Y(u) = \int d^2\Theta\sqrt{h}Y^A\big[N_A(u, x^A) - 3m(u, x^A)D_A C(u, x^A) - D_A m(u, x^A)C(u, x^A)\big],$$

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5. The expression for the charge is not unique, there are other definitions that satisfy all the requirements.

Angular momentum flux problem

1. The flux of angular momentum is the amount of radiated angular between two points, in this case between past and future,

$$\Delta Q_Y = Q_Y[u_2] - Q_Y[u_1] = Q_Y[\infty] - Q_Y[-\infty].$$

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4. The first two modes (Y_0^0 and Y_1^m) of the electric shear are completely arbitrary and even do not appear in the metric or the shear tensor.
5. However they are absolutely essential for the covariance of the formalism. Simply there is no way of consistently eliminitaing those modes.

Angular momentum flux problem

1. In a physical process the news tensor is $\mathcal{O}(G^2)$, so the mass aspect is $\mathcal{O}(G)$.

$$\partial_u m = -\frac{1}{8G} N_{AB} N^{AB} + \frac{1}{4G} D_A D_B N^{AB}.$$

The invariant angular momentum flux is then $\mathcal{O}(G^3)$. The energy flux is also $\mathcal{O}(G^3)$.

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$$\chi^{RR} = -\frac{1}{2}\left(\frac{\partial \chi^{Conserv}}{\partial j} \delta j^{rad} + \frac{\partial \chi^{Conserv}}{\partial E} \delta E^{rad}\right) \propto \mathcal{O}(G^2).$$

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3. Therefore we have to search for an invariant term of order $\mathcal{O}(G^2)$ that gives the correct flux. This is also related to the problem of fixing first two modes of the electric shear.

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2. For the Lorentz group, the associated charge to rotations is angular momentum while the associated charge to the boosts is the position of center of mass.
3. As mentioned above we can not set $C_{(0,0)} = C_{(m,1)} = 0$ because it is simply impossible.
4. We fix these coefficients by making the boost charges zero. This corresponds to placing the center of mass at the origin. This makes sense because a shift in coordinates changes the value of the angular momentum.

$$Q(-\infty)_{Y^{boost}} = 0.$$

5. Since we are studying flux we must specify $C(u, x^A)_{(m, l \leq 1)}$ separately for $u = -\infty$ and $u = \infty$. Two natural choices are
   a) Set $C(\infty, x^A)_{(m, l \leq 1)} = C(-\infty, x^A)_{(m, l \leq 1)}$.
   b) Solve $Q(\infty)_{Y^{boost}} = 0$.

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2. The charges are intrinsic because they are translation invariant.
3. The first two modes of $C(u, x^A)$ are fixed by placing center of mass at the origin.
4. To study the flux we also need to fix the first modes at the future, two natural choices were introduced here.

Thank You