Exotic RG flows from holography

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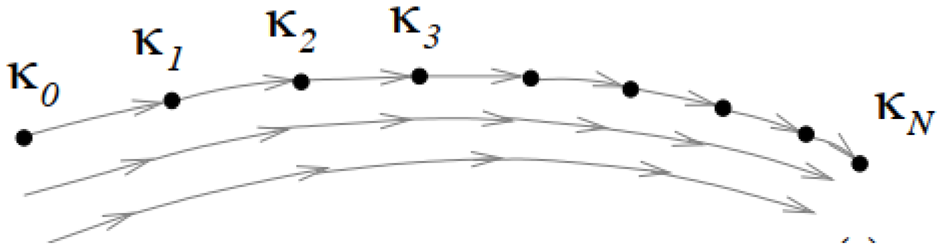
The Wilsonian renormalization group (RG) is a set of transformations on the parameters κ and Λ of $Z[\Lambda]$, that leave the value of $Z[\Lambda]$ exactly unchanged.

$$\boldsymbol{Z[\Lambda] = \int_{\Lambda} D\phi e^{i\int d^4x\,[L(\phi(x))]}} \longrightarrow \boldsymbol{Z[\Lambda] = \int_{\Lambda-\delta\Lambda} D\phi e^{i\int d^4x\,[L(\phi(x))+\delta L(\phi(x))]}}$$

$$A \rightarrow A' \rightarrow A'' \rightarrow \cdots \rightarrow A^{N-1} \rightarrow A^N$$



$$\mu \frac{d}{d\mu} \kappa(\mu) = \beta(g)$$

Quantum Field Theory in a Nutshell. A. Zee

Quantum Field Theory 2 . Perimeter Institute. David, F. (2012).

We consider a d+1 dimension asymptotically AdS space-time with **flat** boundary where QFT lives on.

Bulk theory

$$S = \int d^{d+1}x \sqrt{-g}\,\left(R - \frac{1}{2}\,\partial_a\phi\,\partial^a\phi\, - V\,(\phi)\right)$$

u :
holographic coordinate

$\phi = \phi(u)$:
scalar field coupled to gravitation field

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu$$

QFT energy:

$$\mu = \mu_0 e^{A(u)}$$

$$S = \int d^{d+1}x \sqrt{-g}\,\left(R - \frac{1}{2}\,\partial_a\phi\,\partial^a\phi\; - V\,(\phi)\right)$$

$$2(d-1)\ddot{A}(u) + \partial_u\phi(u)^2 = 0$$

$$d(d-1)\dot{A}^2(u) - \frac{1}{2}\partial_u\phi(u)^2 + V(\phi) = 0$$

→ **Einstein equations**

$$\ddot{\phi} + d\dot{A}\dot{\phi} - V'(\phi) = 0$$

→ **K.G equation**

RG flow for a single coupling *g*:

$$\mu\frac{d}{d\mu}g(\mu) = \beta(g)$$

To make contact with QFT we introduce a new function called as **Superpotential**

$$\boldsymbol{W\big(\phi(u)\big) = -2(d-1)\dot{A}(u)}$$

$$2(d-1)\ddot{A}(u) + \partial_u\phi(u)^2 = 0$$

$$d(d-1)\dot{A}^2(u) - \frac{1}{2}\partial_u\phi(u)^2 + V(\phi) = 0$$

$\longrightarrow$

$$\boldsymbol{W'(\phi) = \partial_u\phi(u)}$$

$$\boldsymbol{\frac{1}{2}W'^2(\phi) = V(\phi) + \frac{d}{4(d-1)}W^2(\phi)}$$

✓ Properties

$$\frac{dW}{du} = \frac{d\phi}{du}\frac{dW}{d\phi} = W'^2 \geq 0$$

In (W, u) plane

$$\frac{1}{2}W'^2(\phi) = V(\phi) + \frac{d}{4(d-1)}W^2(\phi)$$

Critical curve →

$$W_b \equiv \sqrt{-\frac{4(d-1)}{d}V(\phi)}$$

✓ Properties

$$\frac{1}{2}W'^2(\phi) = V(\phi) + \frac{d}{4(d-1)}W^2(\phi):$$

Invariant under $(u.W) \to -(u,W)$

$$W'_{\uparrow} = \sqrt{\frac{d}{2(d-1)}\left(W^2(\phi) - W_b^2(\phi)\right)}$$

$$W'_{\downarrow} = -\sqrt{\frac{d}{2(d-1)}\left(W^2(\phi) - W_b^2(\phi)\right)}$$

✓ **Properties**

$$R_{\mu\nu}R^{\mu\nu} = \frac{d}{16(d-1)^2}\left(16(d+1)V^2 + 8dVW^2 + dW^4\right)$$

To find the desired solution approximately, first we should check the following point:

$$W(\phi(u)) = -2(d-1)\dot{A}(u)$$

$$W'(\phi) = \partial_u \phi(u)$$

$$\frac{1}{2}W'^2(\phi) = V(\phi) + \frac{d}{4(d-1)}W^2(\phi)$$

$$V'(\phi) = W'(\phi)\left(W''(\phi) - \frac{d}{2(d-1)}W(\phi)\right)$$

$$\phi = \phi^* \;\rightarrow\; W'(\phi^*) = 0$$

$$V'(\phi^*) = 0 \;\rightarrow\; W''(\phi^*): finite$$

$$V'(\phi^*) \neq 0 \;\rightarrow\; W''(\phi^*): divergent$$

✓ ϕ^*: **Maximum of the potential**

$$\frac{1}{2}W'^2(\phi) = V(\phi) + \frac{d}{4(d-1)}W^2(\phi)$$

$$V = -\frac{d(d-1)}{l^2} + \frac{m^2}{2}\phi^2, \qquad m^2 < 0$$

$$W_+ = \frac{1}{l^2}\left[2(d-1) + \frac{\Delta_+}{2}\phi^2 + O\left(\phi^3\right)\right]$$

$$W_- = \frac{1}{l^2}\left[2(d-1) + \frac{\Delta_-}{2}\phi^2 + O\left(\phi^3\right)\right] + C\phi^{\frac{d}{\Delta_-}}(1 + O(\phi))$$

✓ ϕ^*: Maximum of the potential

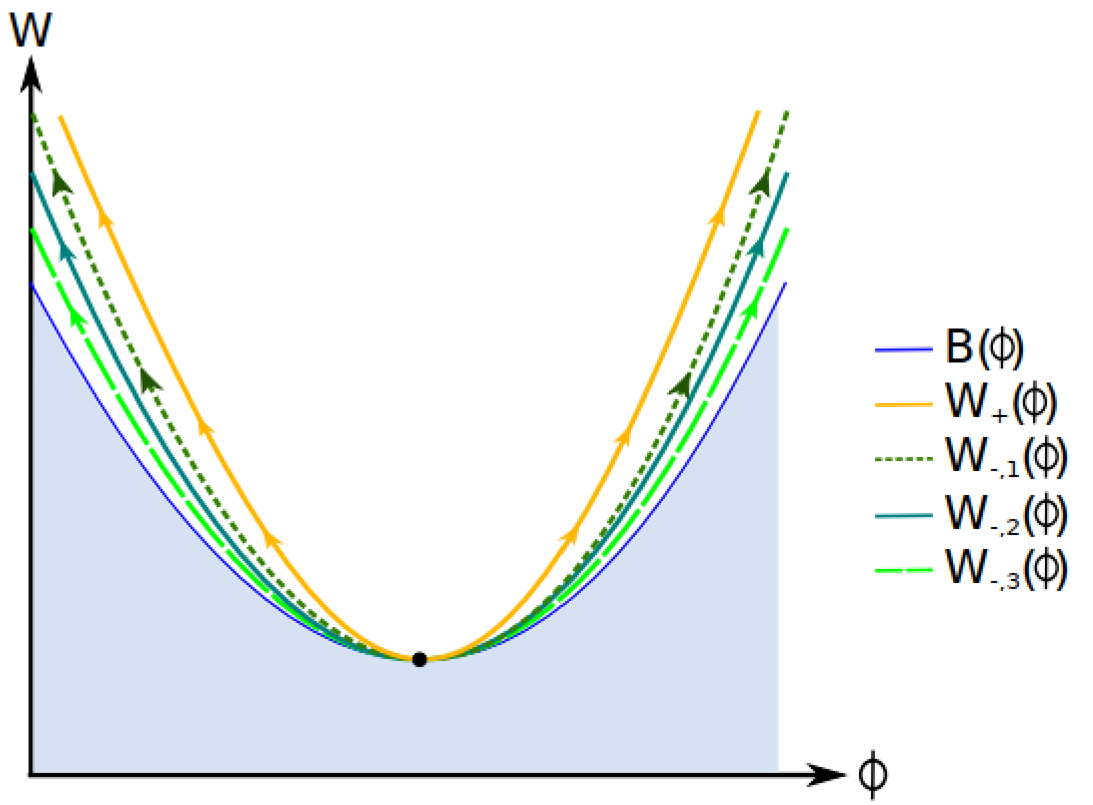
$$V = -\frac{d(d-1)}{l^2} + \frac{m^2}{2}\phi^2, \qquad m^2 < 0$$

$$W_+ = \frac{1}{l^2}\left[2(d-1) + \frac{\Delta_+}{2}\phi^2 + O(\phi^3)\right]$$

$$W_- = \frac{1}{l^2}\left[2(d-1) + \frac{\Delta_-}{2}\phi^2 + O(\phi^3)\right] + C\phi^{\frac{d}{\Delta}}(1 + O(\phi))$$

$$B(\phi) = W_b \equiv \sqrt{-\frac{4(d-1)}{d}V(\phi)}$$

$$\Delta_\pm = \frac{1}{2}\left(d \pm \sqrt{d^2 + 4m^2 l^2}\right) \to m^2 \geq -\frac{d^2}{4l^2}$$



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✓ ϕ^*: Maximum of the potential

$$W(\phi(u)) = -2(d-1)\dot{A}(u) \qquad W'(\phi) = \partial_u \phi(u)$$

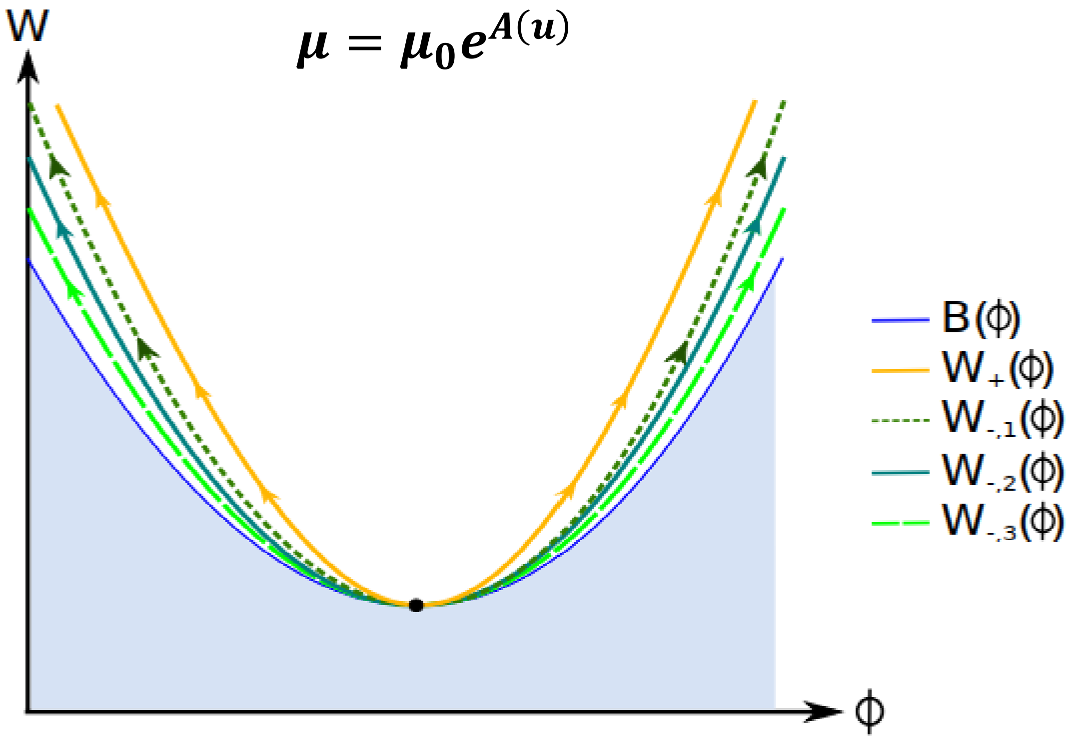
$$\phi_\uparrow = \phi_+ e^{\frac{\Delta_+ u}{l}} + \cdots$$

$$\phi_\downarrow = \phi_- e^{\frac{\Delta_- u}{l}} + \frac{dCl}{\Delta_-(2\Delta_+ - d)} \phi_-^{\frac{\Delta_+}{\Delta_-}} e^{\frac{\Delta_+ u}{l}}$$

$$A_\uparrow = -\frac{u-u_*}{l} + \frac{\phi_+^2}{8(d-1)} e^{\frac{2\Delta_+ u}{l}}$$

$$A_\downarrow = -\frac{u-u_*}{l} + \frac{\phi_-^2}{8(d-1)} e^{\frac{2\Delta_- u}{l}}$$

$$\mu = \mu_0 e^{A(u)}$$



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$$A_\uparrow = -\frac{u-u_*}{l} + \frac{\phi_+^2}{8(d-1)} e^{\frac{2\Delta_+ u}{l}}$$

$$A_\downarrow = -\frac{u-u_*}{l} + \frac{\phi_-^2}{8(d-1)} e^{\frac{2\Delta_- u}{l}}$$

$$\langle \mathcal{O} \rangle_{W_-} = \frac{1}{\ell^{\Delta_+}} \frac{d}{\Delta_-} C\ell\, \phi_-^{\Delta_+/\Delta_-}$$

$$\langle \mathcal{O} \rangle_{W_+} = \frac{2\Delta_+ - d}{\ell^{\Delta_+}} \phi_+ .$$

$$J = \frac{\phi_-}{l^{\Delta_-}}$$

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✓ **ϕ^*: Minimum of the potential**

$$W_\pm = \frac{1}{l^2}\left[2(d-1) + \frac{\Delta_\pm}{2}\phi^2 + O(\phi^3)\right]$$

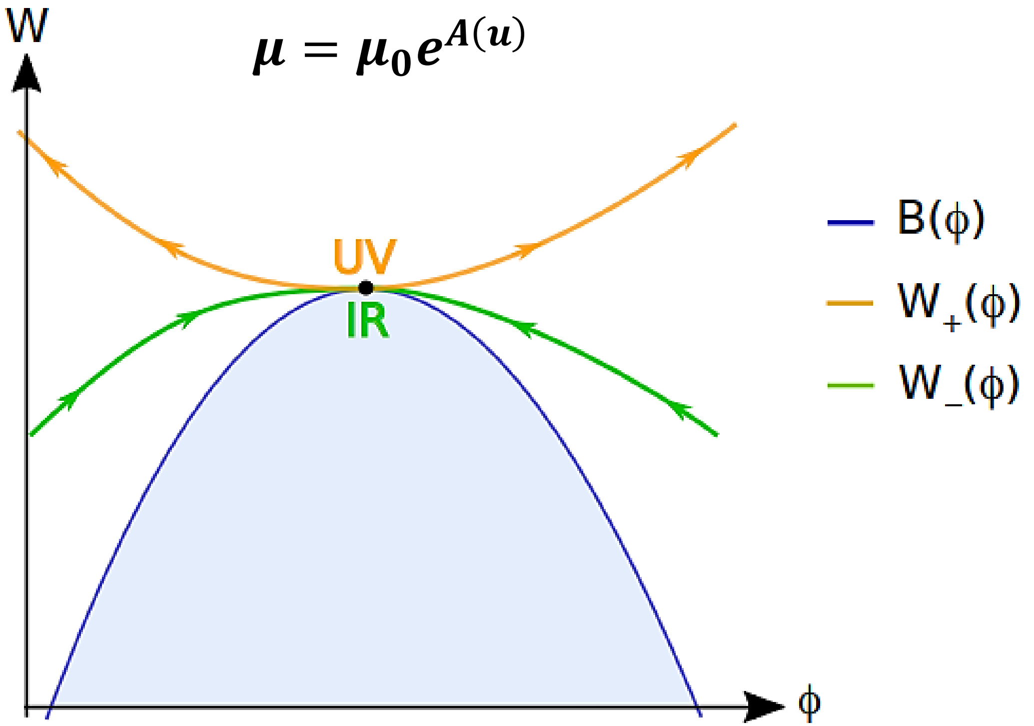
$$\phi_\uparrow = \phi_+ e^{\frac{\Delta_+ u}{l}} + \cdots$$

$$\phi_\downarrow = \phi_- e^{\frac{\Delta_- u}{l}}$$

$$A_\pm = -\frac{u-u_*}{l} - \frac{\phi_\pm^2}{8(d-1)} e^{\frac{2\Delta_\pm u}{l}}$$

$$V = -\frac{d(d-1)}{l^2} + \frac{m^2}{2}\phi^2, \qquad m^2 > 0$$

$$\mu = \mu_0 e^{A(u)}$$



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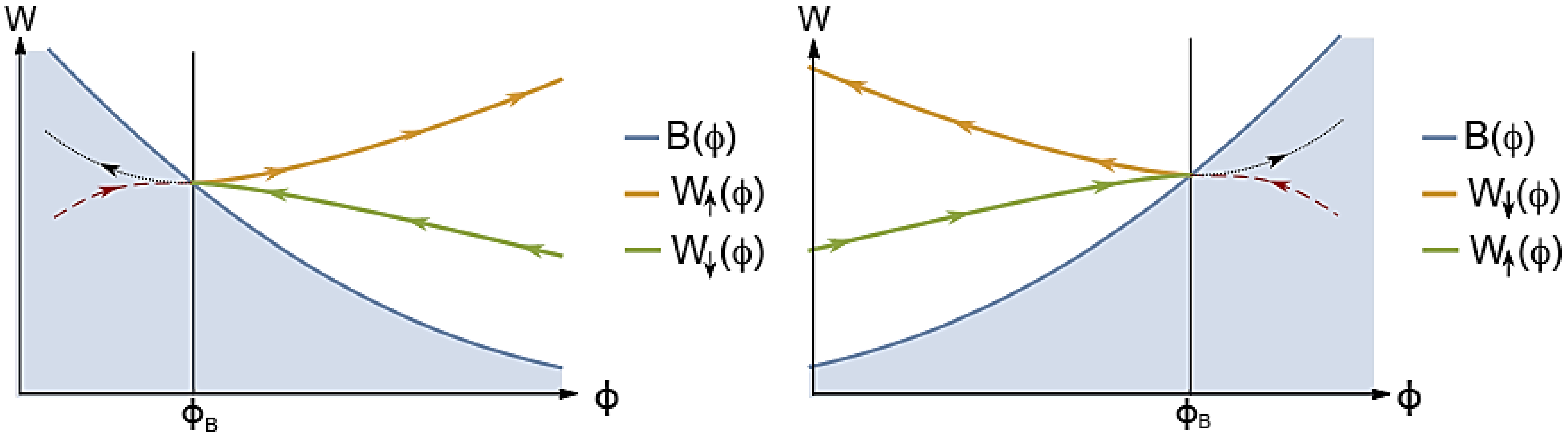
✓ Bounces!



(a) (b)

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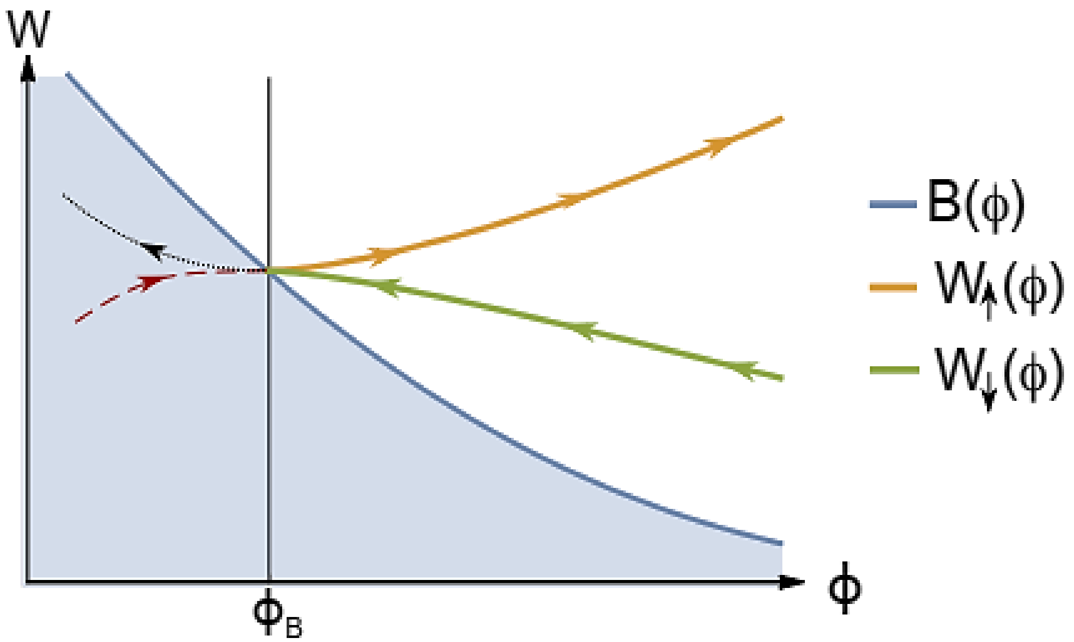
✓ **Bounces:** $V'(\phi_b) \neq 0 \;\rightarrow\; W''(\phi_b)\text{: divergent}$

$$V'(\phi) = W'(\phi)\left(W''(\phi) - \frac{d}{2(d-1)}W(\phi)\right)$$

$$W'(\phi) \approx \pm\sqrt{2(\phi-\phi_b)V'(\phi_b)}$$

$$W_\downarrow \approx W_B - \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi-\phi_b)^{\frac{3}{2}}$$

$$W_\uparrow \approx W_B + \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi-\phi_b)^{\frac{3}{2}}$$



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✓ **Bounces:** $V'(\phi_b) \neq 0 \;\; \rightarrow \; W''(\phi_b): divergent$

$$\mu = \mu_0 e^{A(u)}$$

$$W'(\phi) = \partial_u \phi(u) \qquad W\big(\phi(u)\big) = -2(d-1)\dot{A}(u)$$

$$W_\downarrow \approx W_B - \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi - \phi_b)^{\frac{3}{2}}$$

$$W_\uparrow \approx W_B + \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi - \phi_b)^{\frac{3}{2}}$$

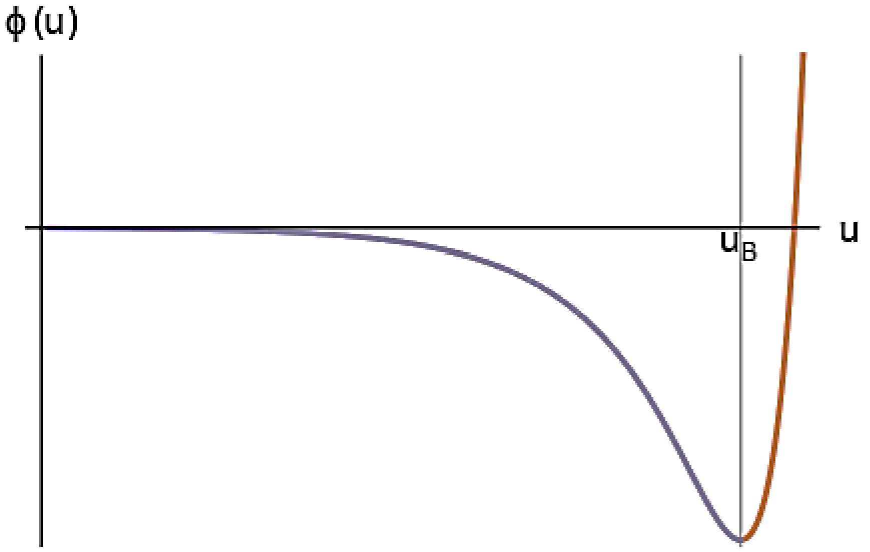
$$\beta(\phi) = \frac{d\phi}{d\log\mu} = -2(d-1)\frac{W'(\phi)}{W(\phi)}$$

✓ **Bounces:** $V'(\phi_b) \neq 0 \rightarrow W''(\phi_b)$: *divergent*

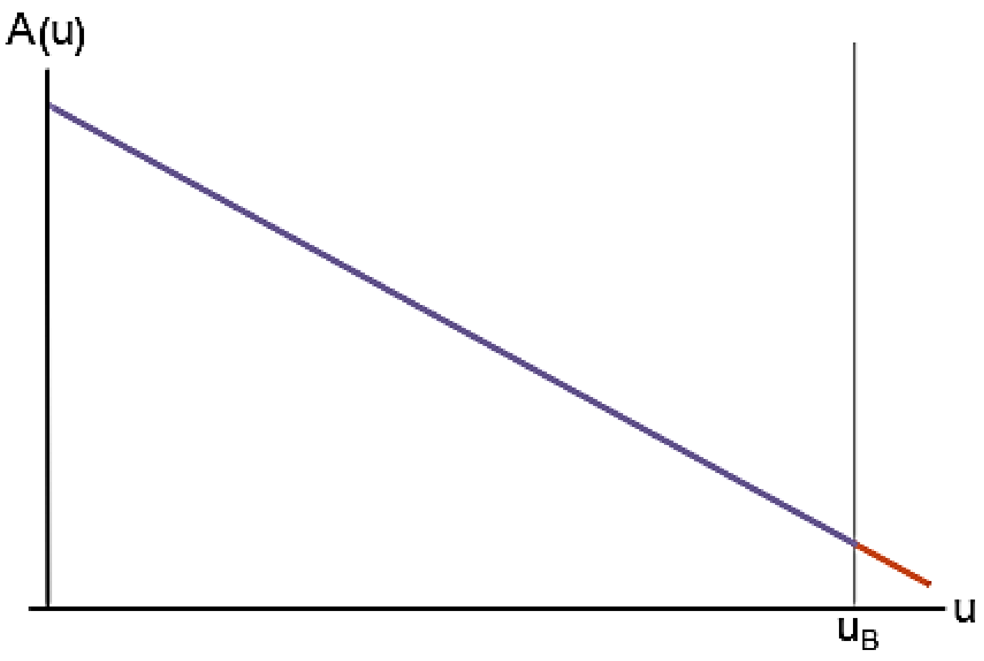
$$W_{\downarrow} \approx W_B - \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi - \phi_b)^{\frac{3}{2}}$$

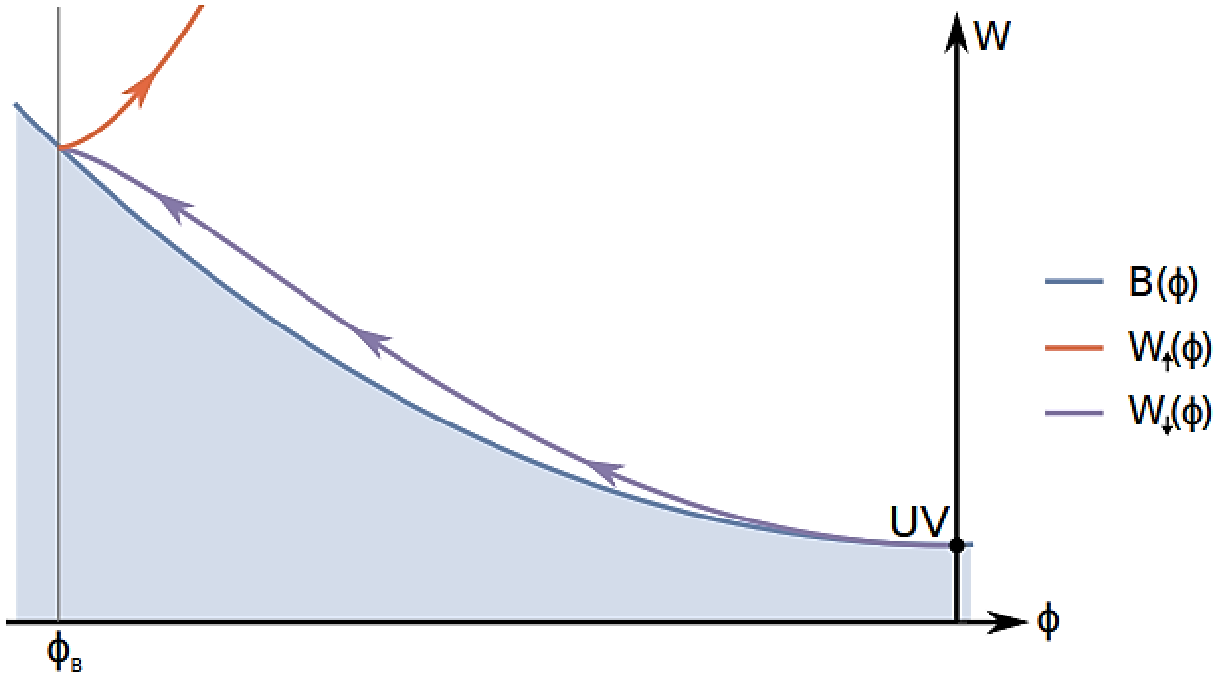
$$W_{\uparrow} \approx W_B + \frac{2}{3}\sqrt{2V'(\phi_b)}(\phi - \phi_b)^{\frac{3}{2}}$$

$$\phi(u) = \phi_B + \frac{V'(\phi_B)}{2}(u-u_B)^2 + \mathcal{O}(u-u_B)^3 = \begin{cases} \phi_{\uparrow}(u) \text{ for } u > u_B, \\ \phi_{\downarrow}(u) \text{ for } u < u_B. \end{cases}$$



- **Bounces:** $V'(\phi_b) \neq 0 \;\; \rightarrow \; W''(\phi_b): divergent$





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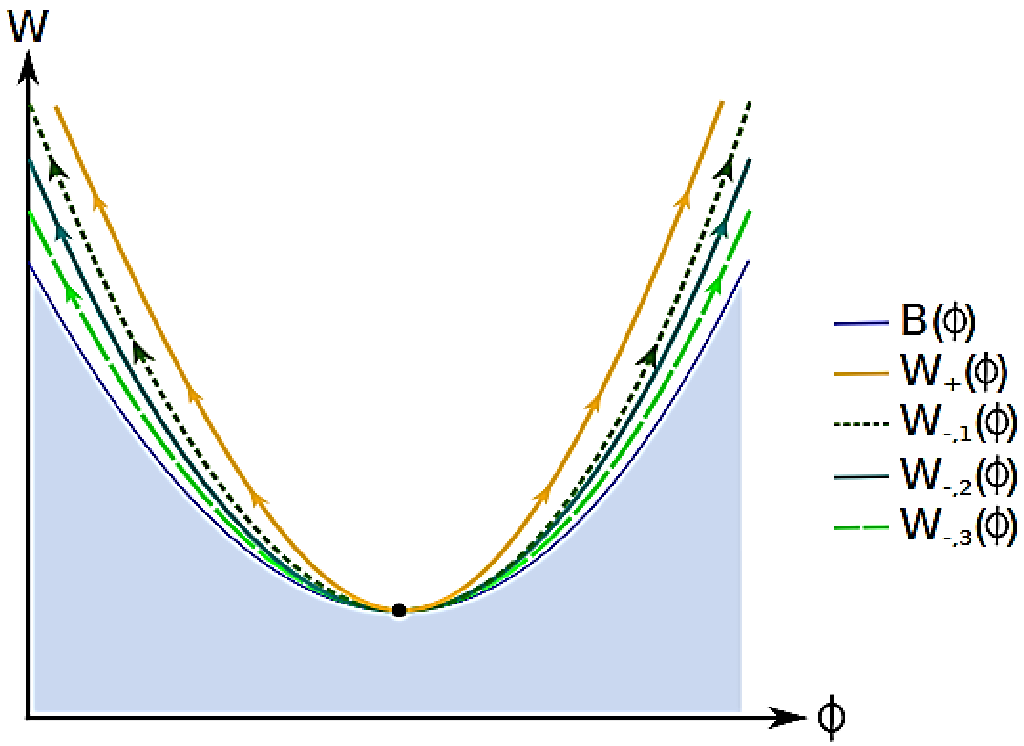

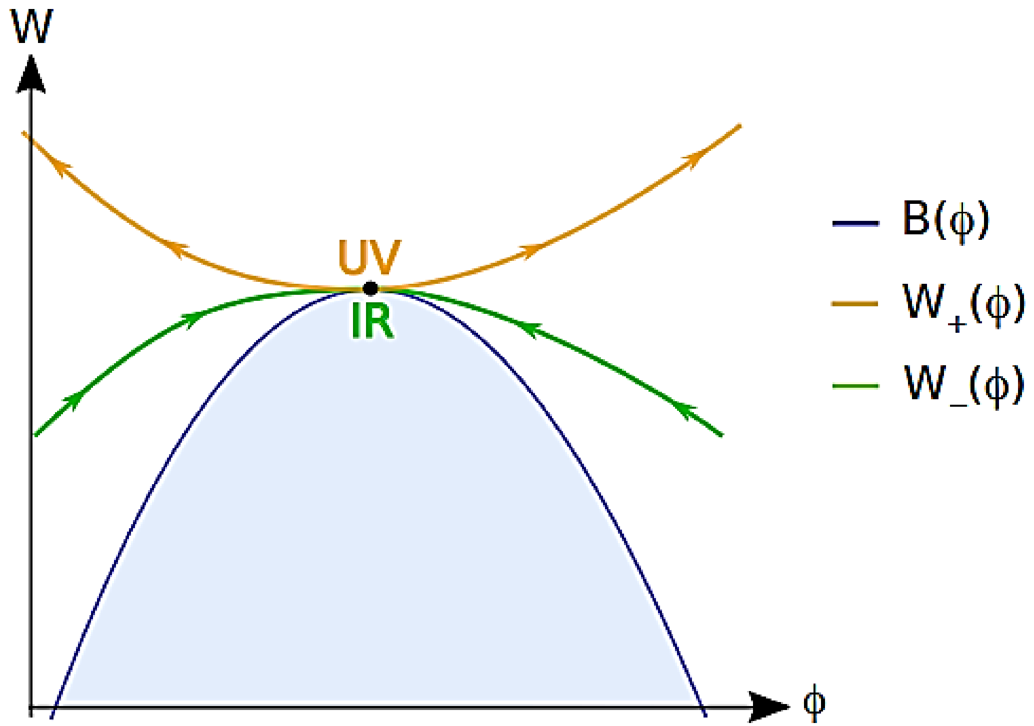

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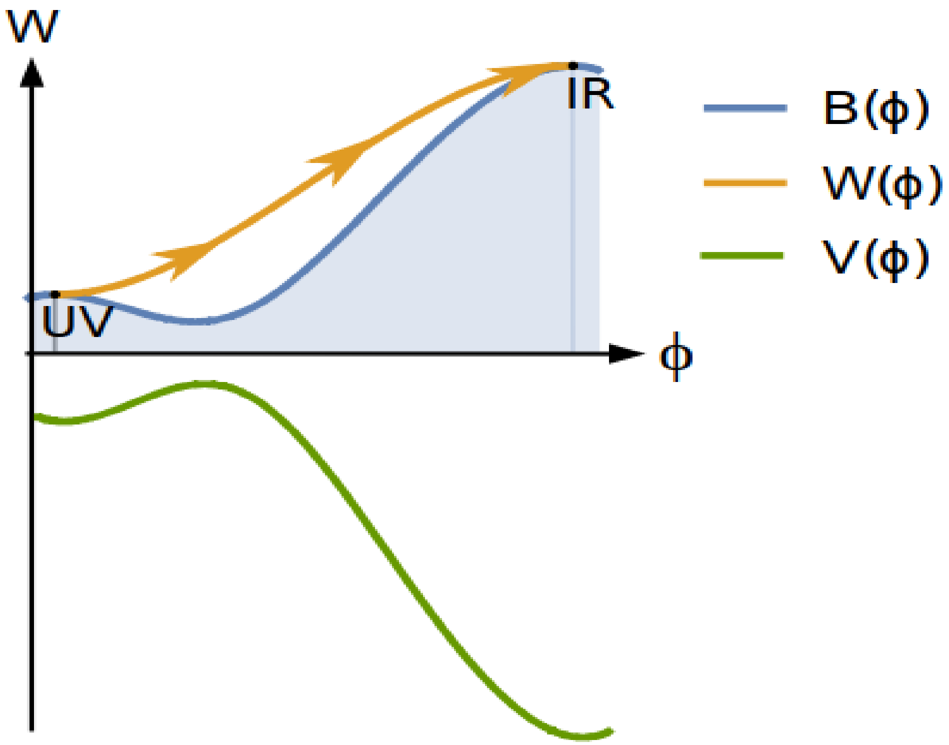
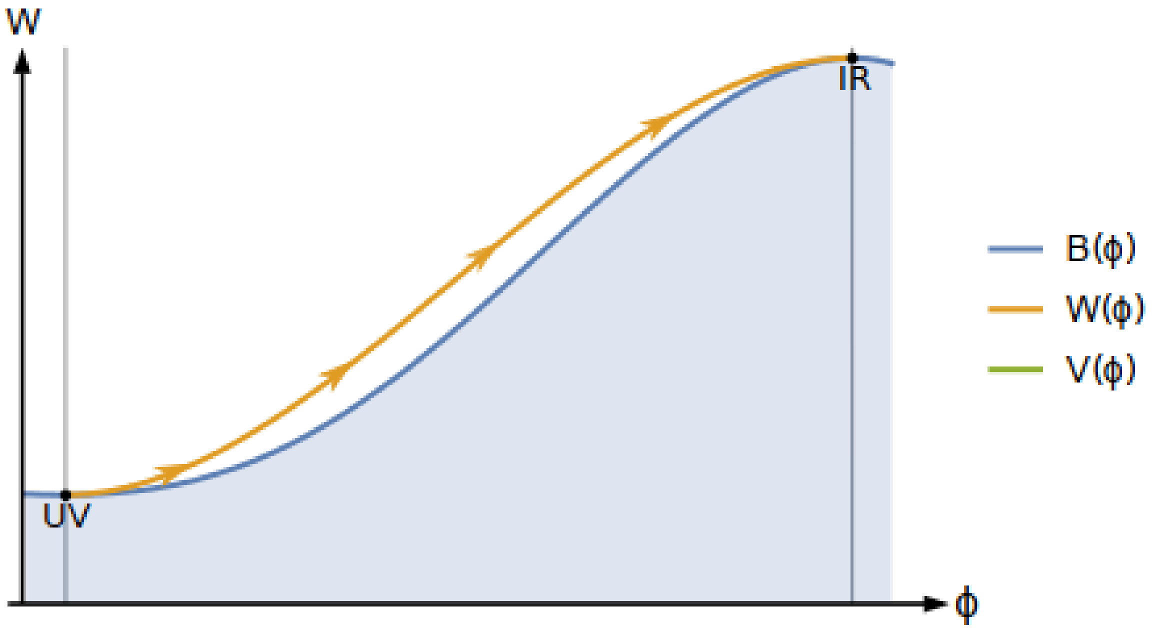
✓ Potential with <u>two</u> minima

$$V(\phi) = \frac{k\nu}{2}\left(1-\left(\frac{\phi}{\nu}\right)^2\right)^2 - \frac{d}{4(d-1)}\left(k\nu^2\left(\frac{\phi}{\nu}\right)\left(1-\frac{1}{3}\left(\frac{\phi}{\nu}\right)^2\right)+W_0\right)^2$$





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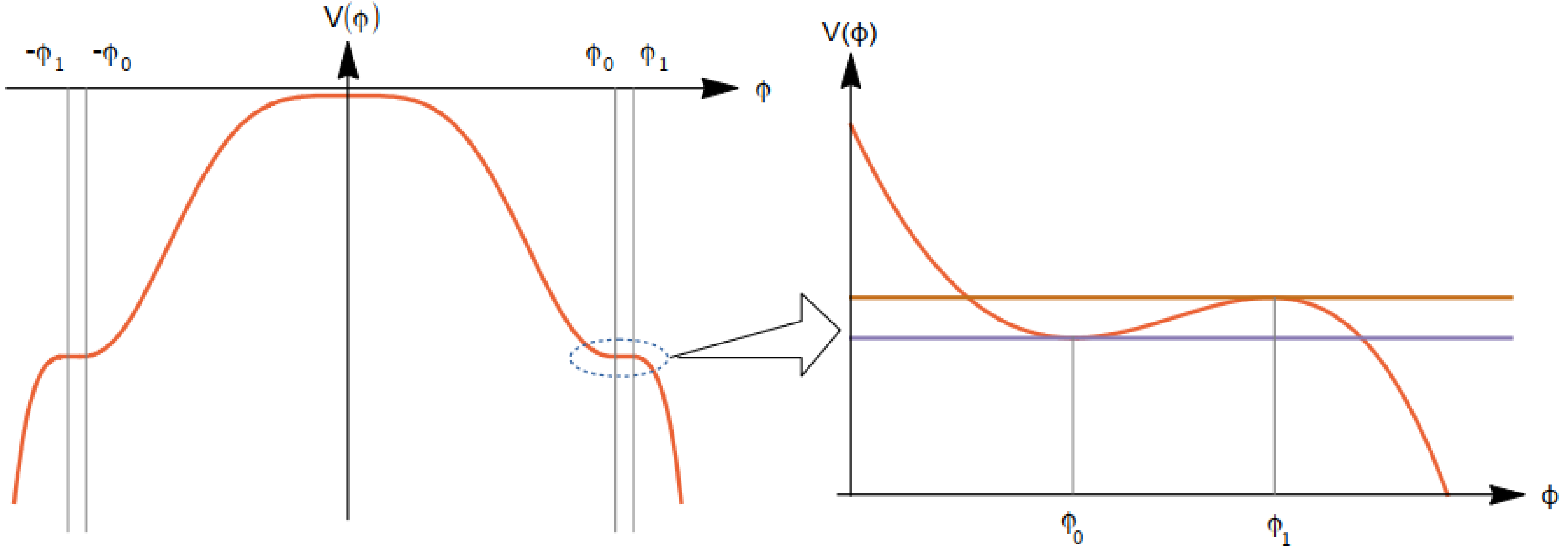
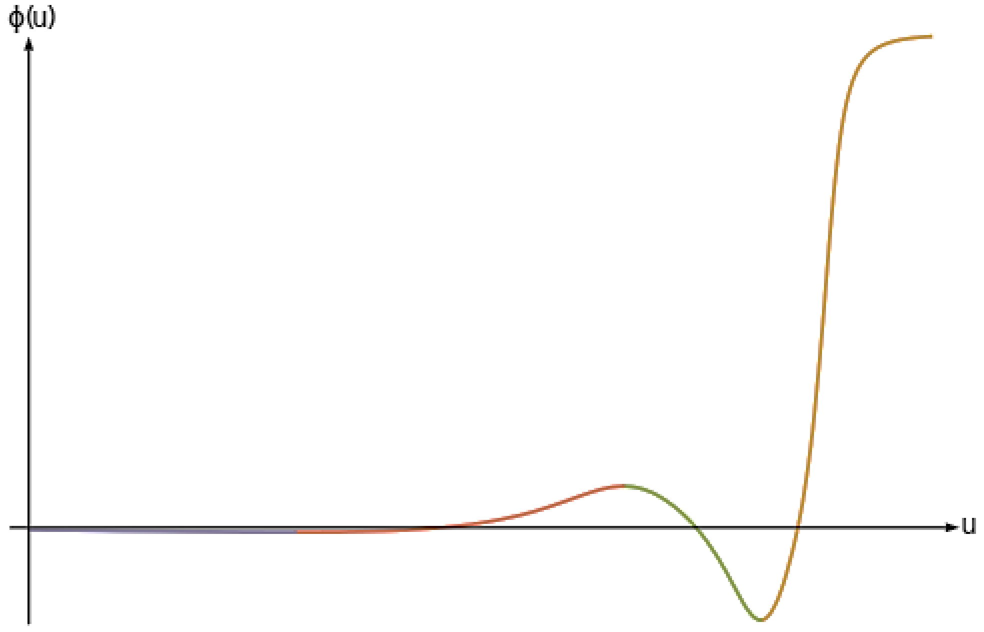
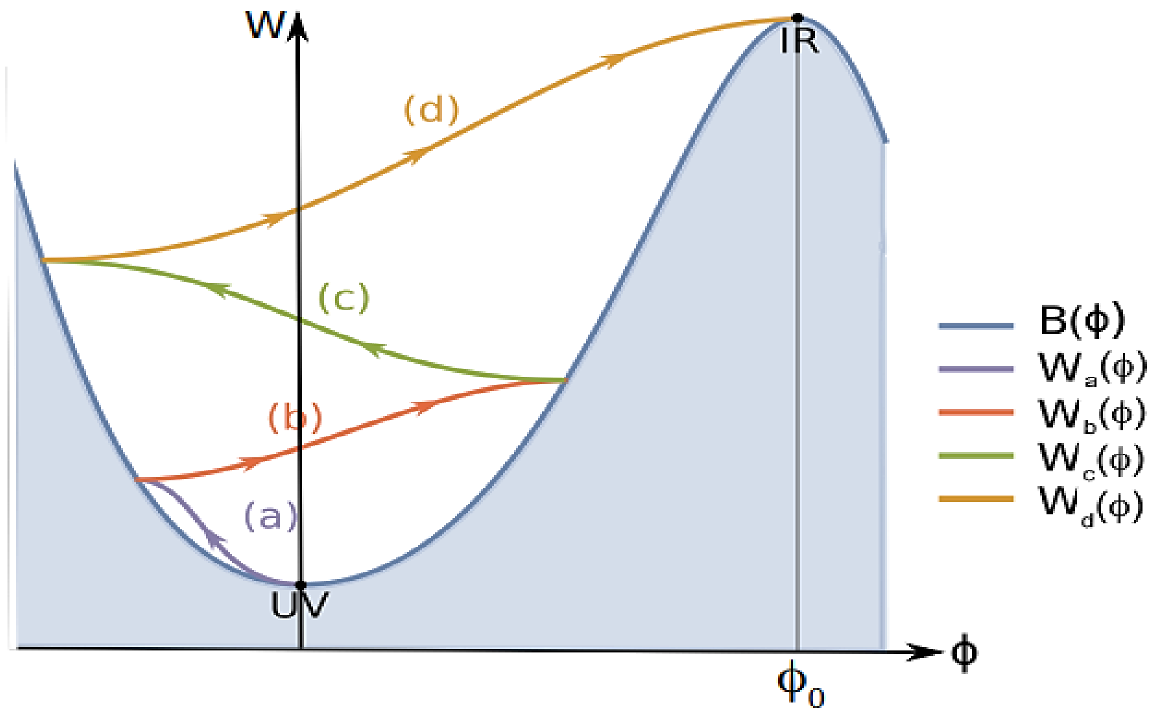
✓ Bounding solutions



✓ Bouncing solutions





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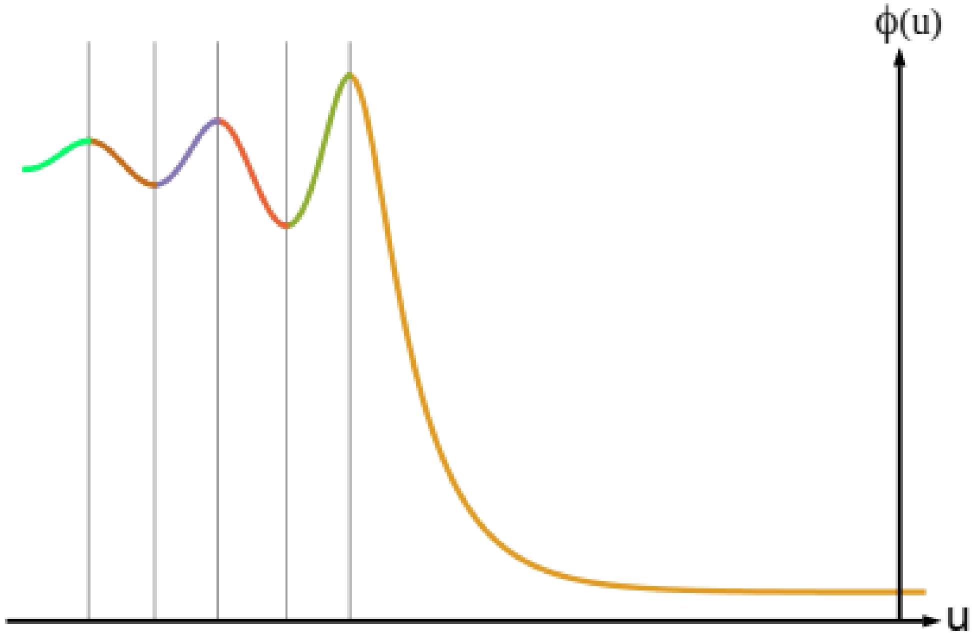
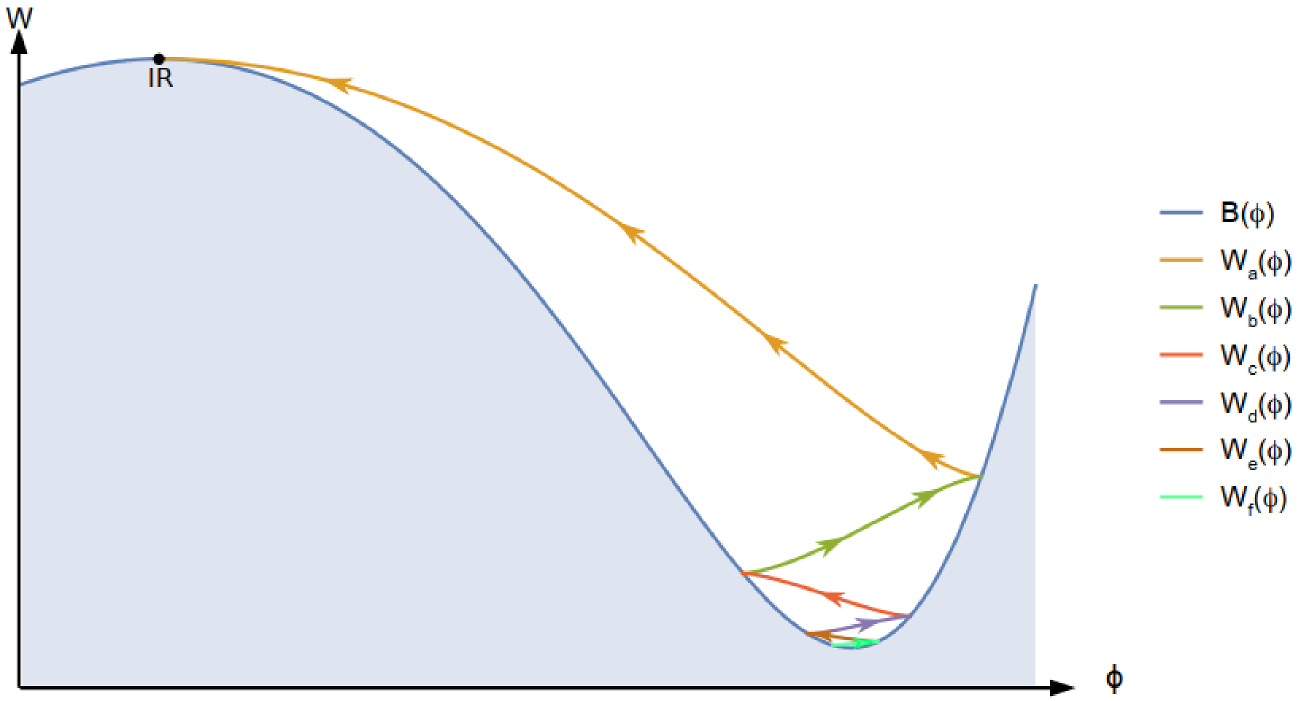
✓ Cascading solutions





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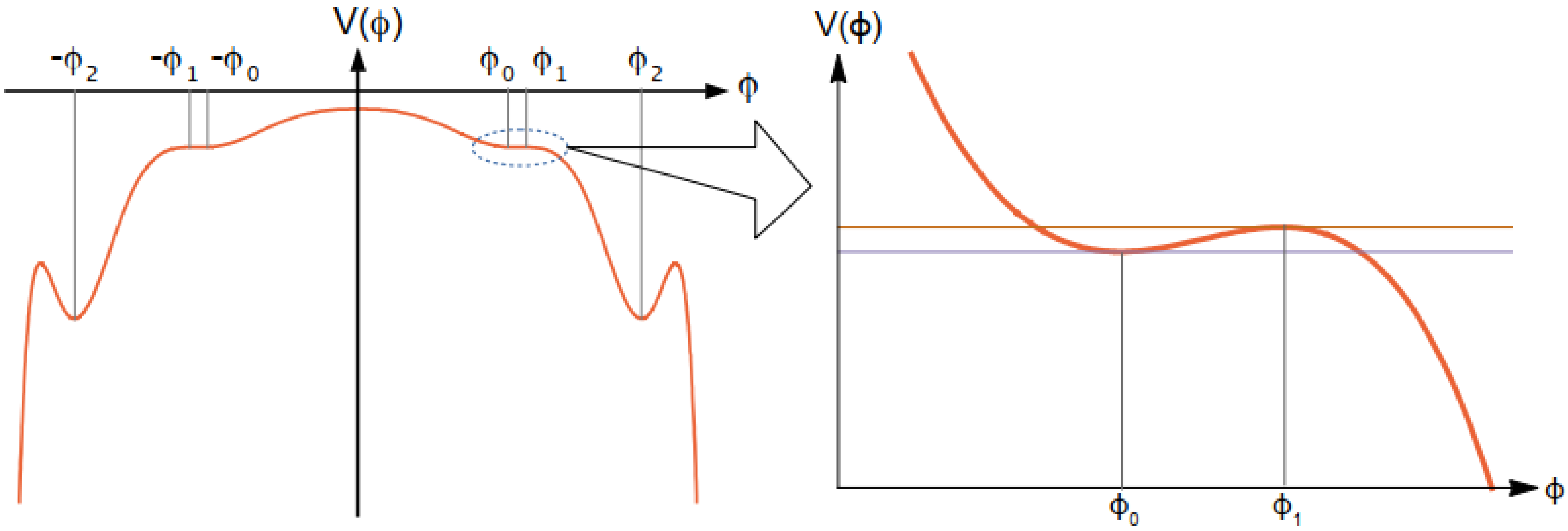
✓ Solutions skipping fixed points



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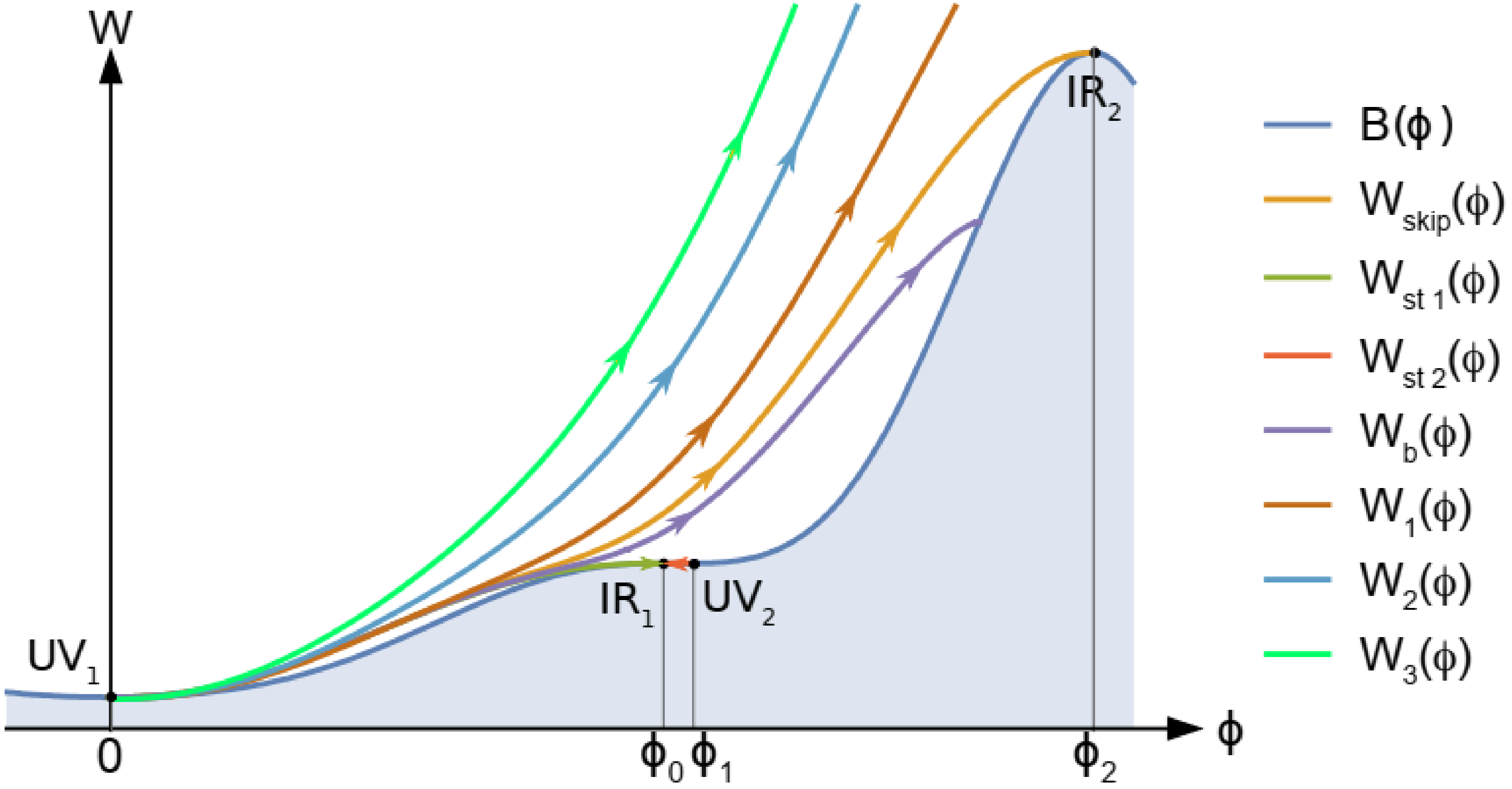

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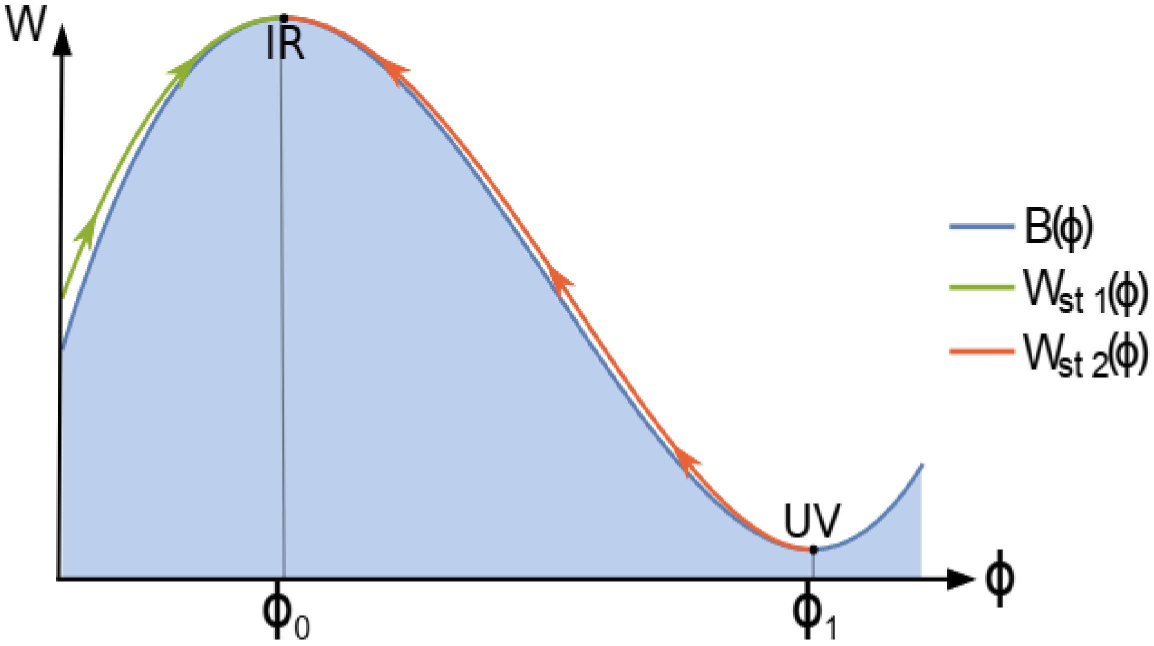

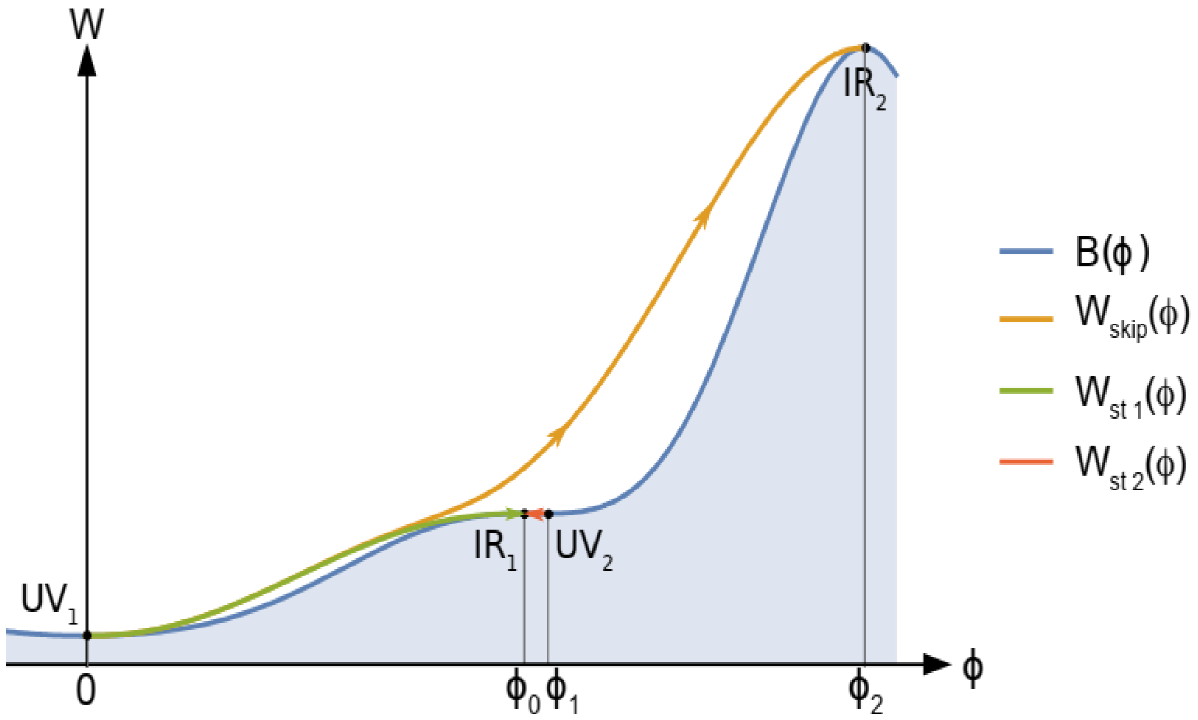

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✓ **Far from boundary!** $\phi \to +\infty$

$$W^2 - Q^2 W'^2 = B^2 \to Q = \sqrt{\frac{2(d-1)}{d}}$$

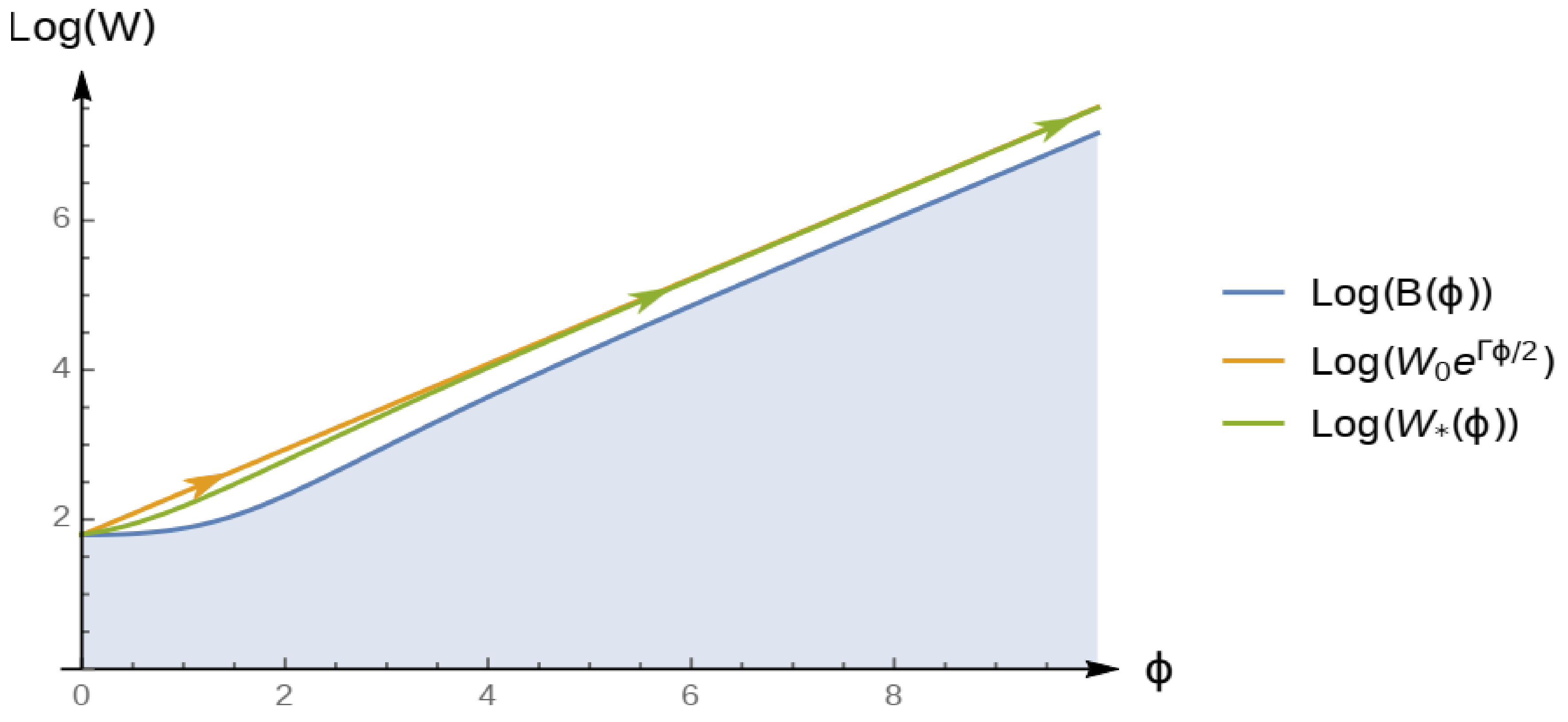
Continuous branch

$$W(\phi) \approx C \exp\left(\frac{1}{Q}\phi\right)$$

Special solution

$$W(\phi) \approx \alpha B(\phi) \to \quad \alpha = \sqrt{\frac{1}{1-k^2Q^2}} \quad \& \quad k = \lim_{\phi \to +\infty} \frac{1}{B}\frac{dB}{d\phi}$$

✓ **Far from boundary! $\phi \to +\infty$**



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✓ **Far from boundary!** $\phi \to -\infty$

$$W^2 - Q^2 W'^2 = B^2 \to Q = \sqrt{\frac{2(d-1)}{d}}$$

$$W \to W_0 = B_0, \quad \phi \to \infty$$

$$W > 0, \qquad W' > 0$$

$$\delta W \simeq C \exp\left(\frac{1}{Q}\int \frac{W}{W'}\right)$$

✓ **True Vacuum!**

$$F = S_E[\phi_-]$$

$$\langle \mathcal{O} \rangle_{W_-} = \frac{1}{\ell^{\Delta_+}} \frac{d}{\Delta_-} C\ell\, \phi_-^{\Delta_+/\Delta_-}$$

$$S_{on-shell} = C - C_{ct} \int d^d\, x \phi_-^{\frac{d}{(d-\Delta)}}$$

$$F_i = (C_i - C_{ct}) \int d^d\, x \phi_-^{\frac{d}{(d-\Delta)}}$$